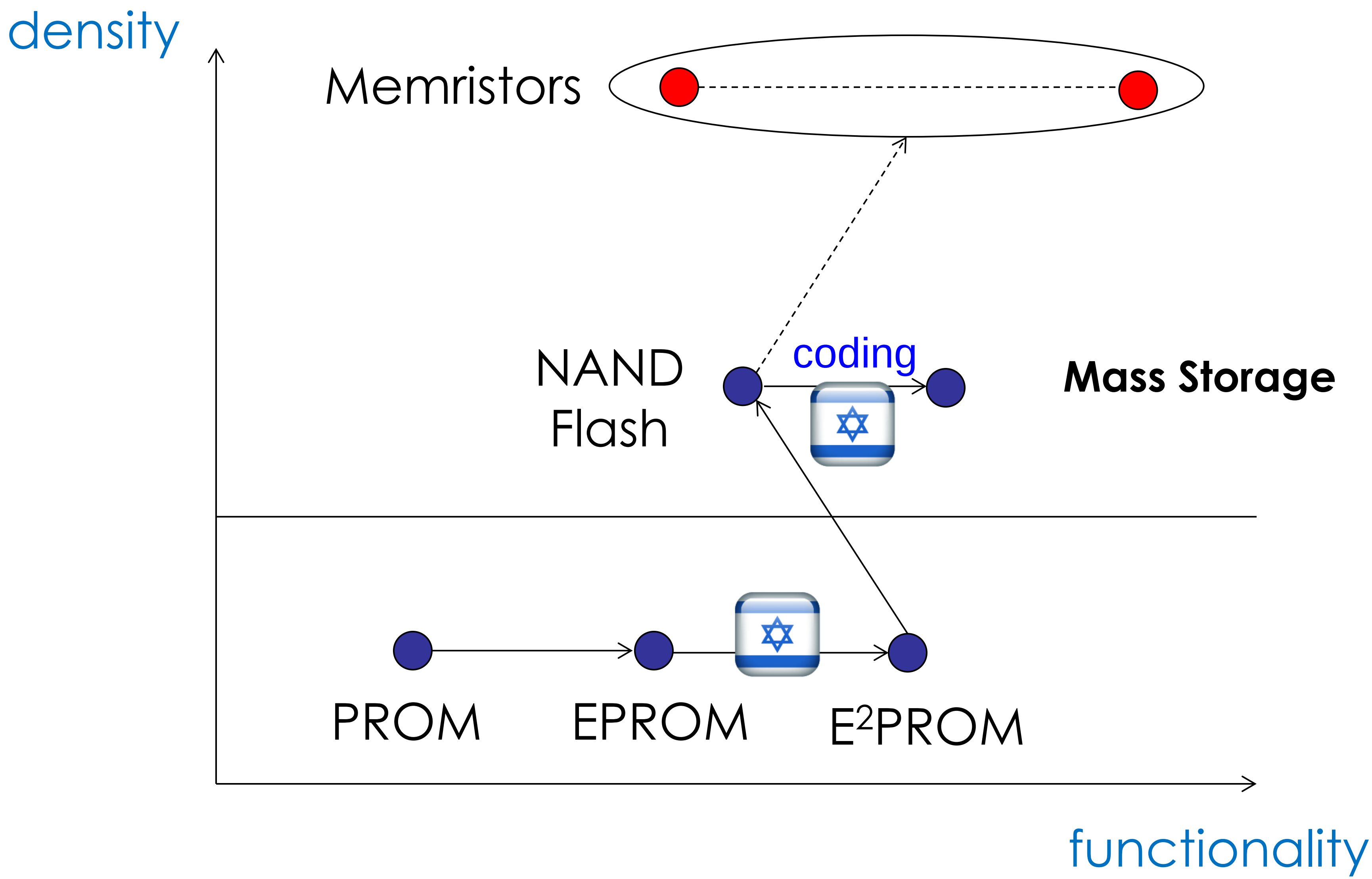
Electronics
Computers
Communications

Seminar on Coding for Non-Volatile Memories

236803/048704 – CS/EE Departments, Technion

MEMRISTORS / RESISTIVE MEMORIES

Non-Volatile Memory Progression

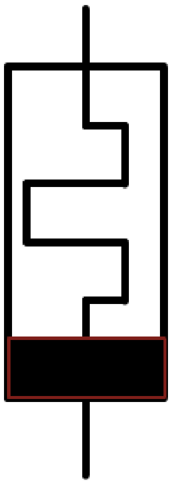


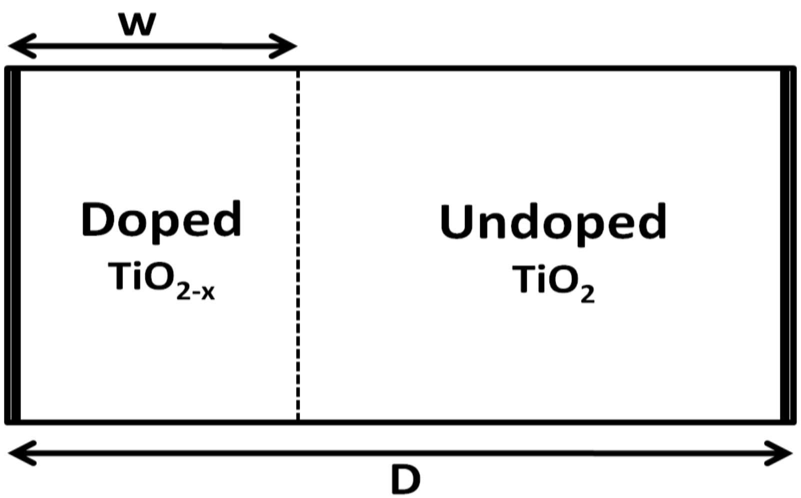
Outline

- Memristors
  - Promising storage technology
- Sneak Paths
  - Main functional limitation
- Coding for Sneak-Path mitigation
  - Sneak path elimination
  - Sneak path as random error source

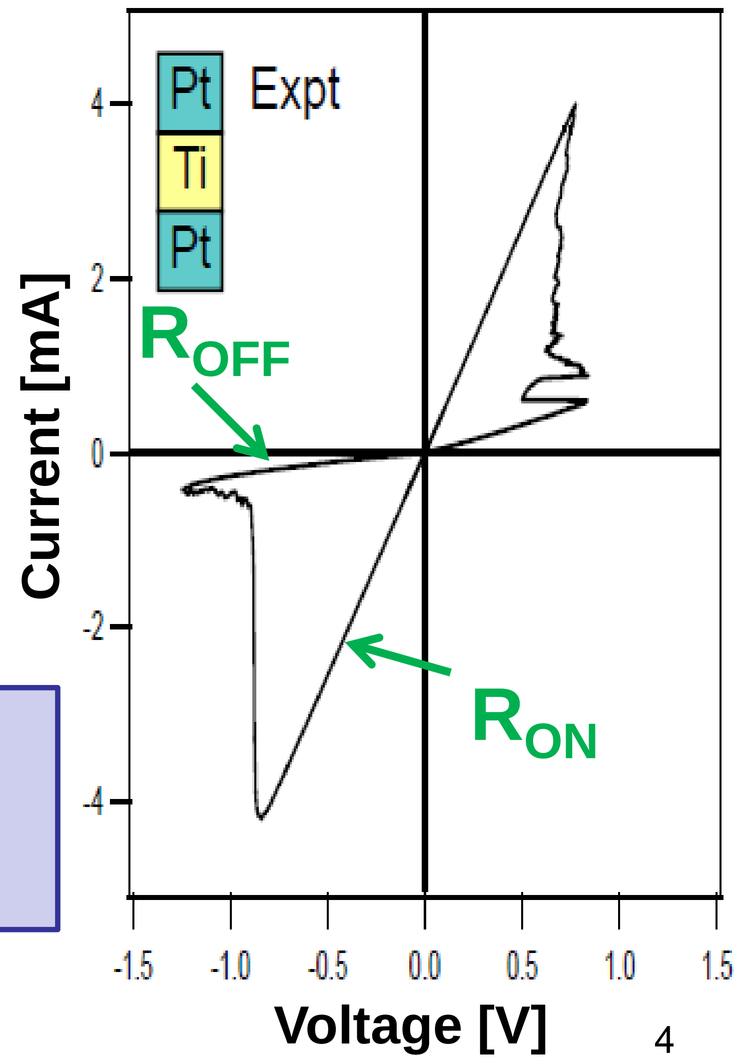
Memristors

- 2008 Hewlett Packard



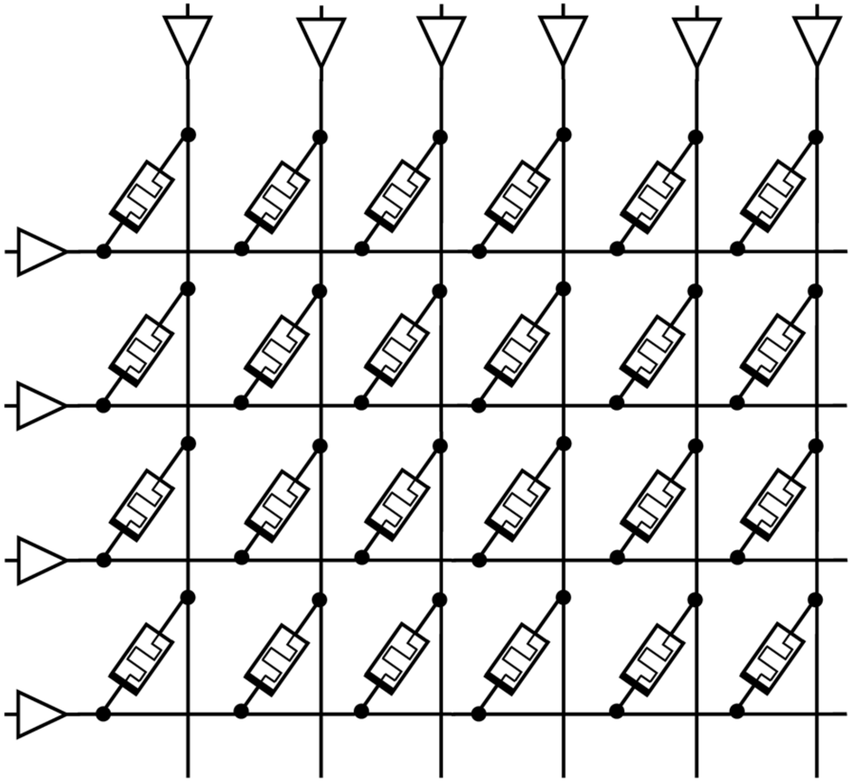


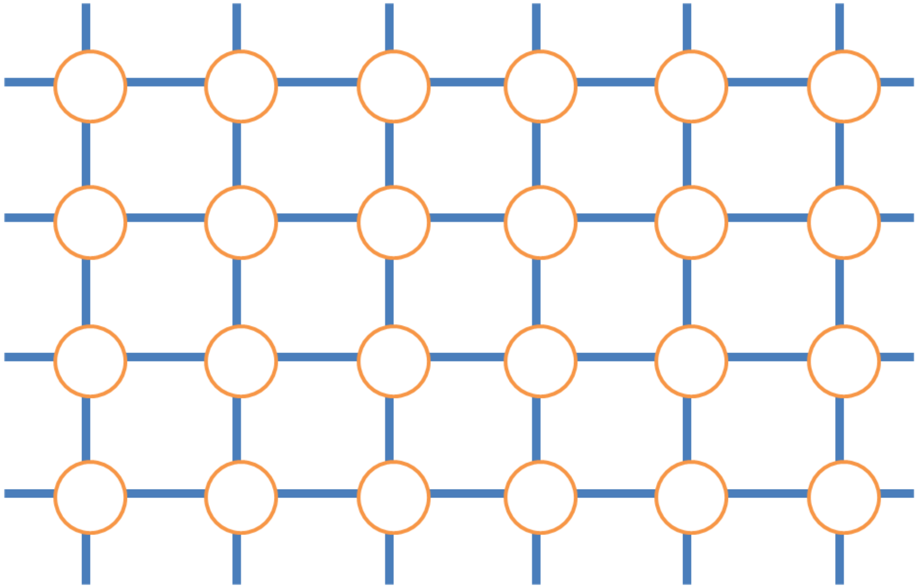
$$M(q) = R_{OFF}\left(1 - \frac{\mu_v R_{ON}}{D^2} q(t)\right)$$

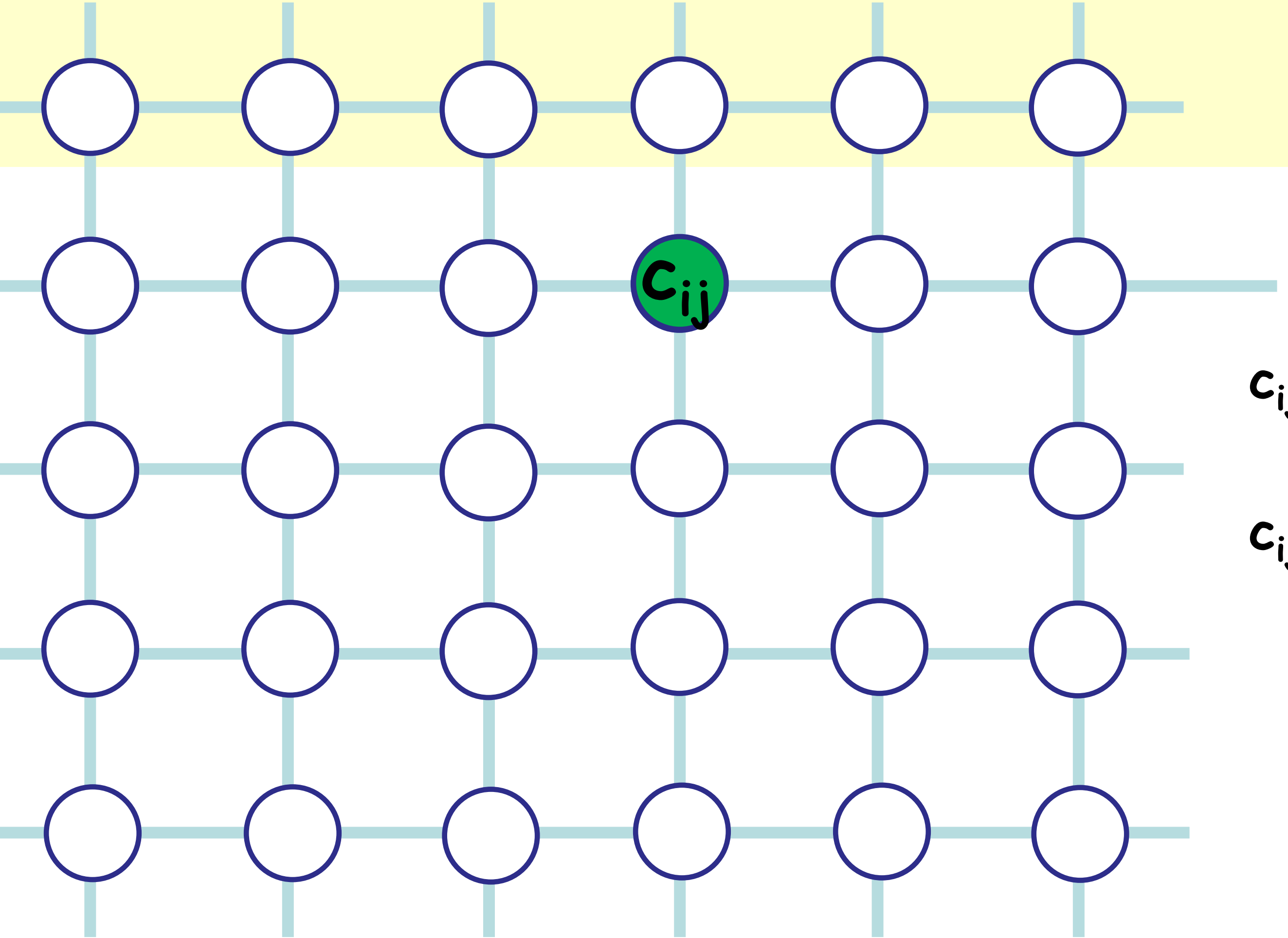


D.B. Strukov et al, “The missing memristor found,” Nature, 2008

Crossbar Arrays

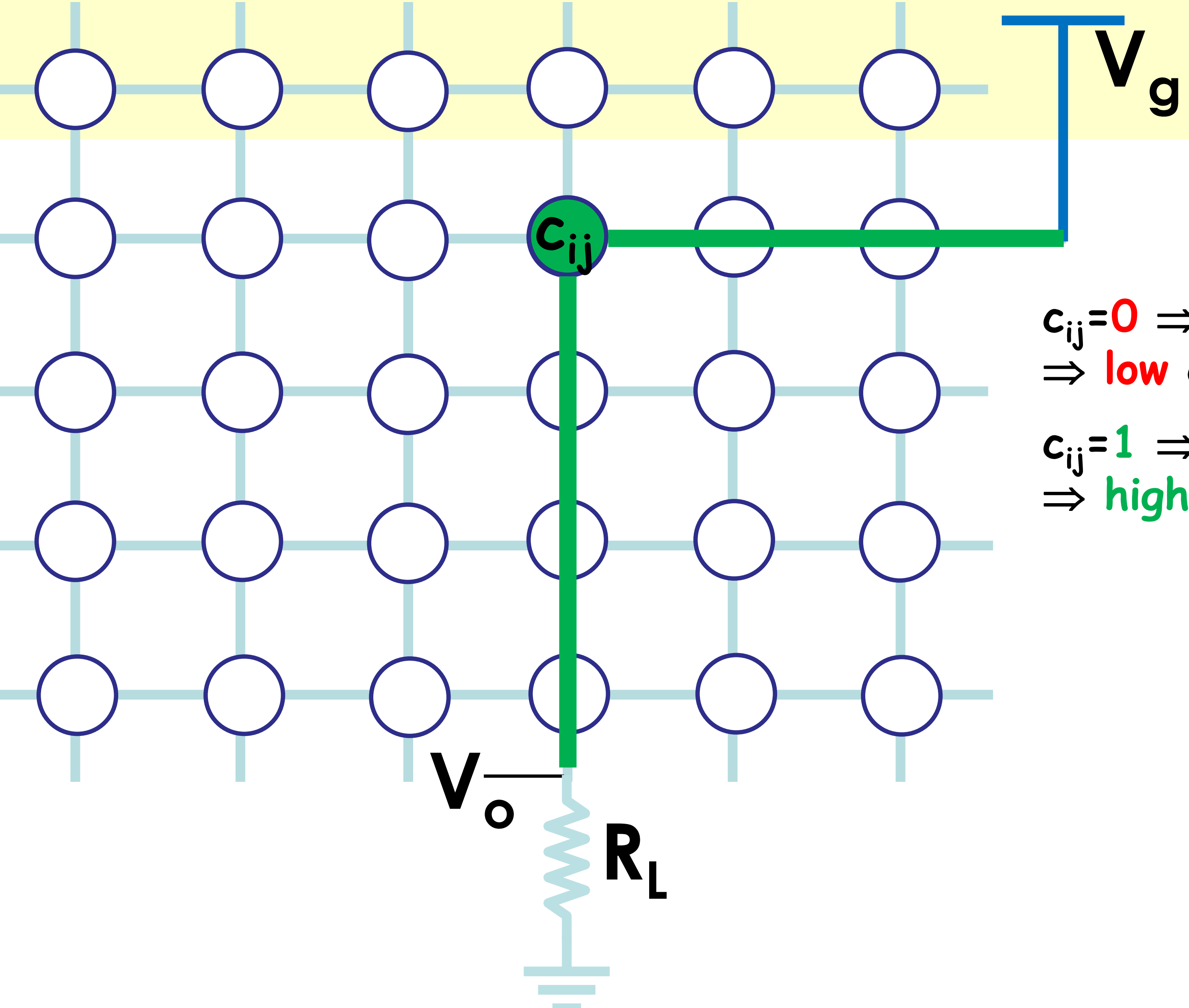






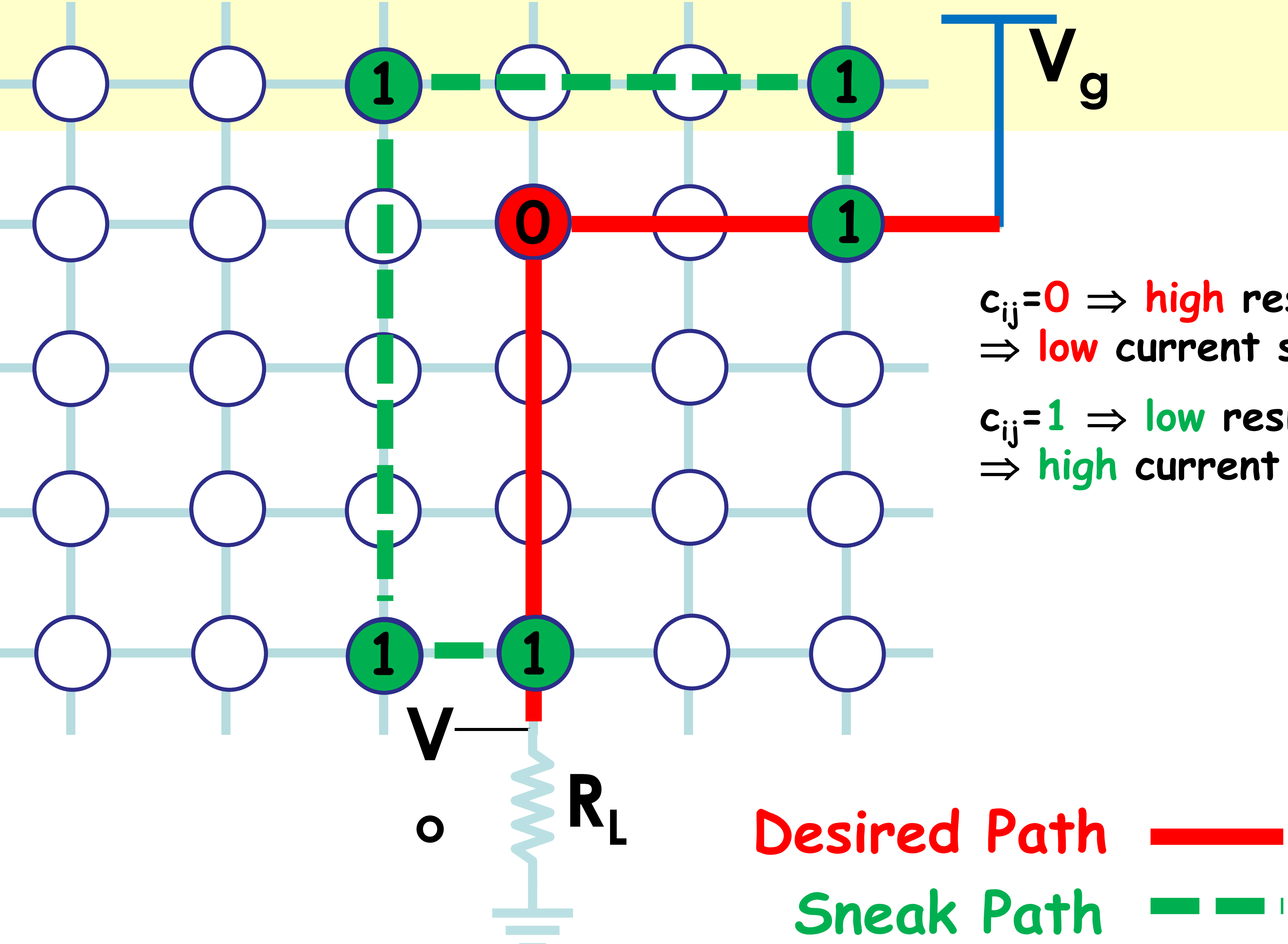
$c_{ij}=0 \Rightarrow$ high resistance

$c_{ij}=1 \Rightarrow$ low resistance



$c_{ij}=0$ ⇒ high resistance ⇒ low current sensed

$c_{ij}=1$ ⇒ low resistance ⇒ high current sensed



$c_{ij}=0 \Rightarrow$ high resistance

$\Rightarrow$ low current sensed

$c_{ij}=1 \Rightarrow$ low resistance

$\Rightarrow$ high current sensed

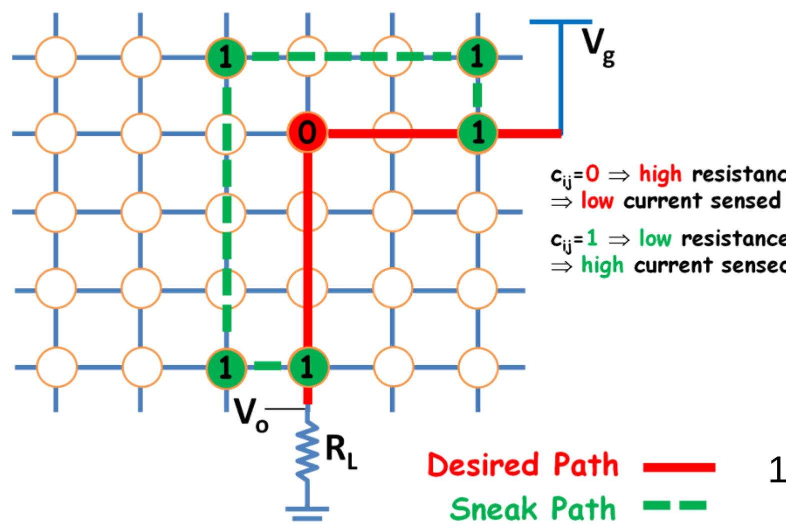
Part 1

Sneak path **elimination**

Sneak Path

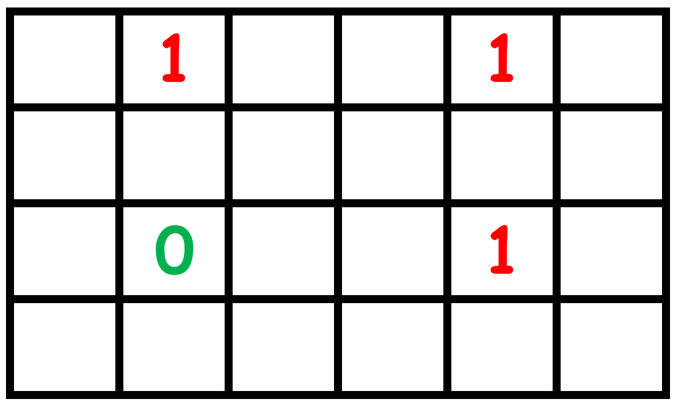
- An array **A** has a **sneak path** of length **2k+1** affecting the **(i,j)** cell if
  - $a_{ij}=0$
  - There exist $r_1,\ldots,r_k$ and $c_1,\ldots,c_k$ such that
    $a_{ic_1} = a_{r_1c_1} = a_{r_1c_2} = \cdots = a_{r_kc_k} = a_{r_kj} = 1$
- An array A satisfies the **sneak-path constraint** if it has no sneak paths and then is called a **sneak-path free array**

		1			1
			0		1
		1	1		

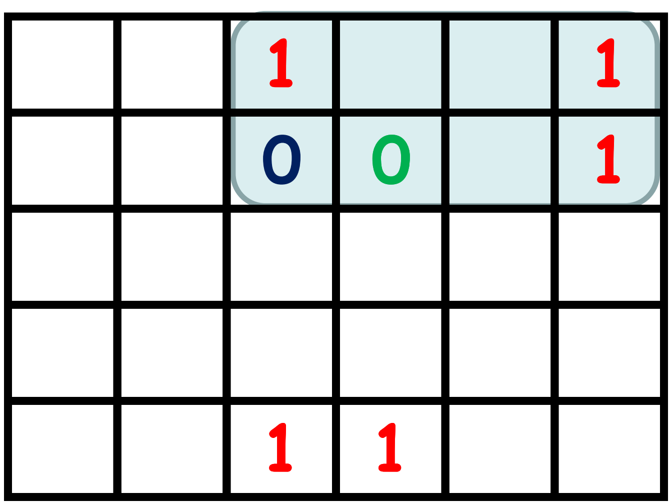


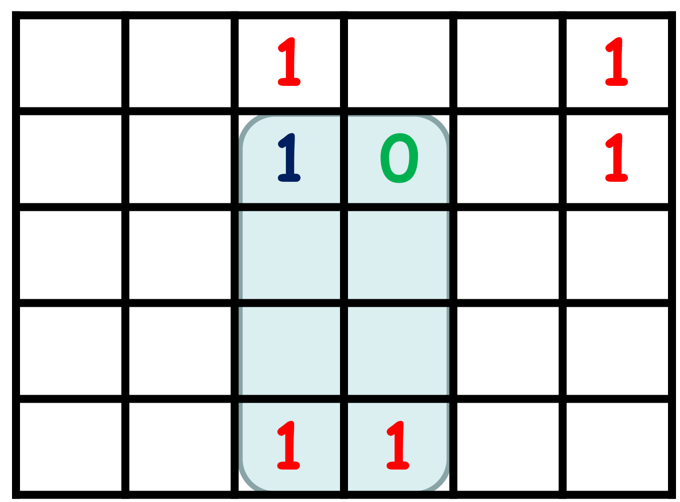
Characterization of Sneak Paths

- **Theorem**: An array **A** has a **sneak path** if and only if it has an **isolated zero-rectangle**



- **Proof:**





Row/Column Overlap Condition

- <u>Lemma</u>:

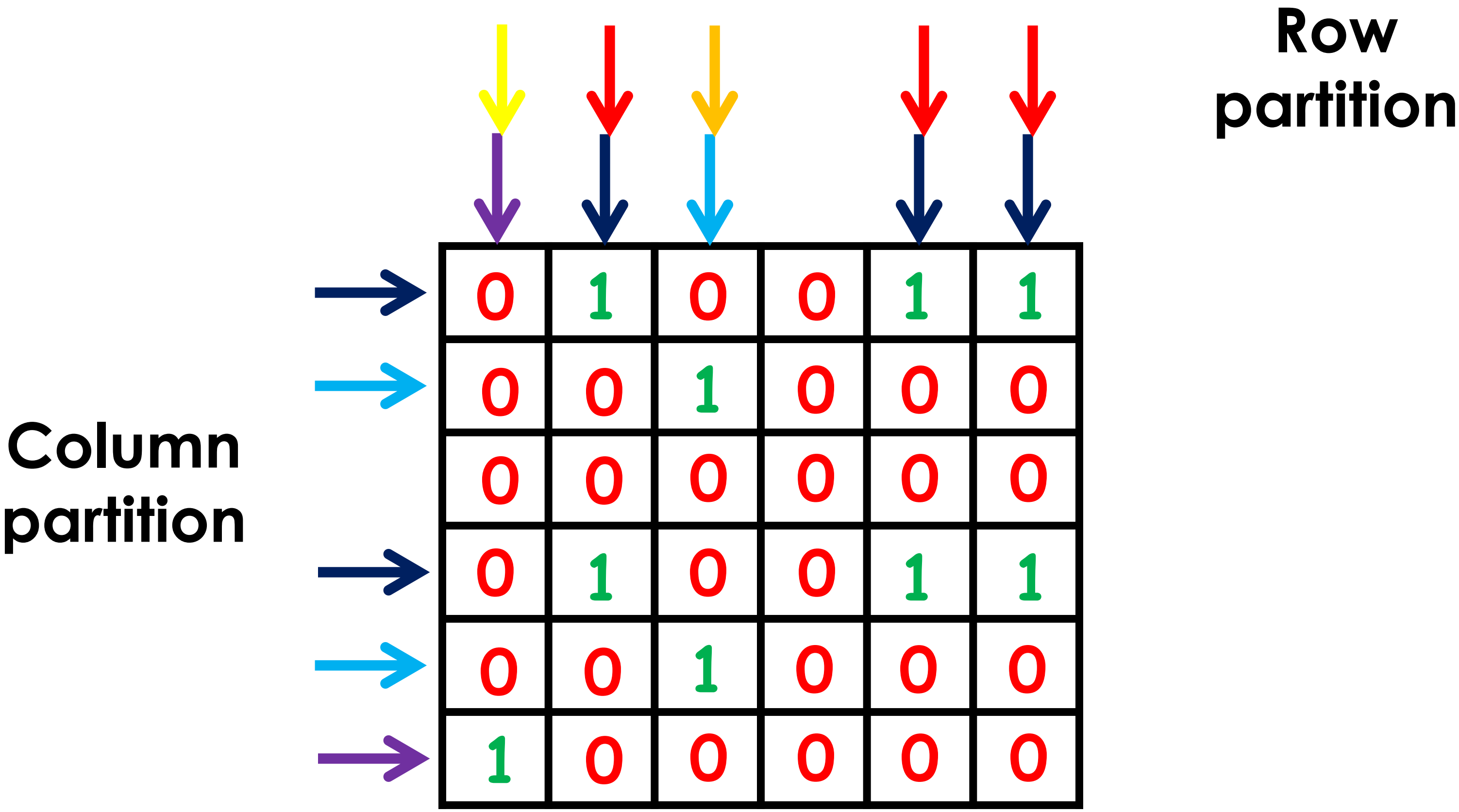
An array has **no isolated zero-rectangles** iff the 1s in every two rows/columns either **completely overlap** **or are** **disjoint**.

Disjoint

0	1	0	0	1	1
0	0	1	0	0	0
0	0	0	0	0	0
0	1	0	0	1	1
0	0	1	0	0	0
1	0	0	0	0	0

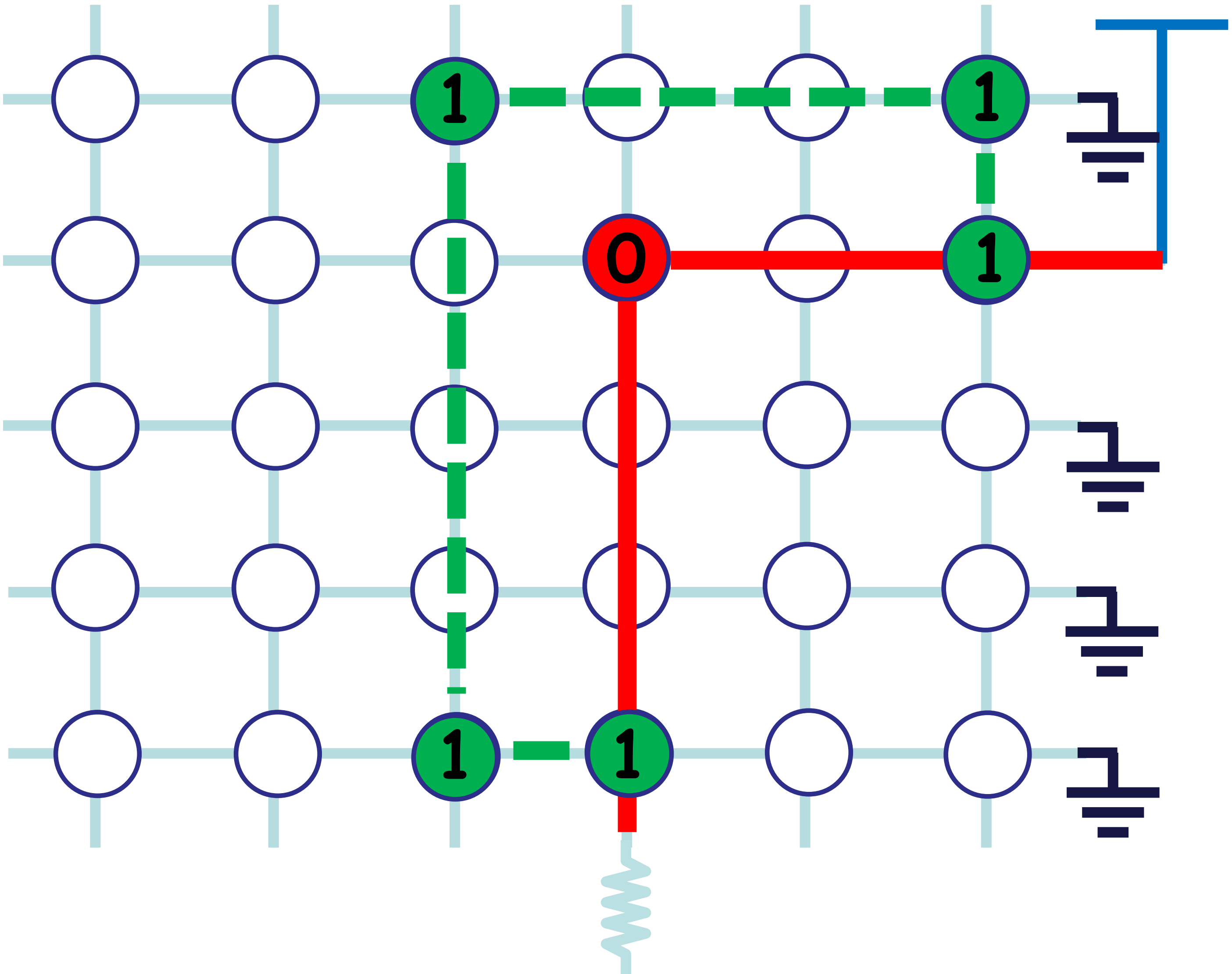
Complete overlap

Encoding Sneak-Path Free Arrays

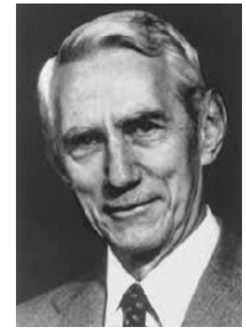


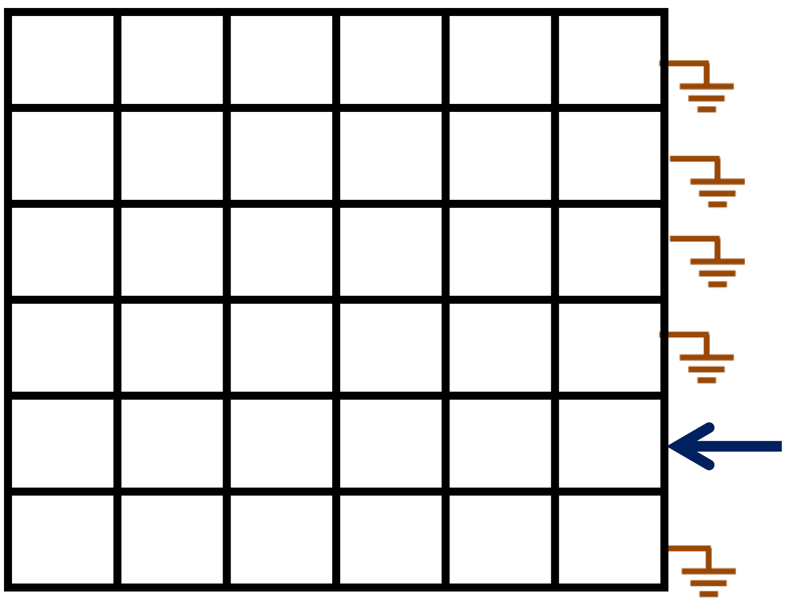
≈2n log n arrays, when n=m

Grounding – an EE Solution



EE or IT?



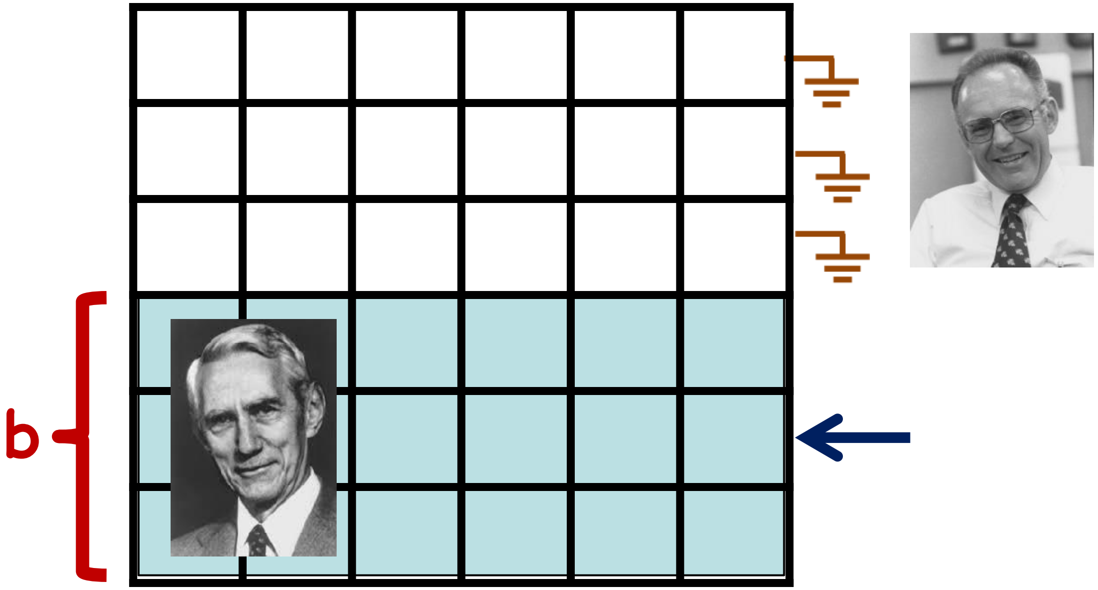


0	1	0	0	1	1
0	0	1	0	0	0
0	0	0	0	0	0
0	1	0	0	1	1
0	0	1	0	0	0
1	0	0	0	0	0

High read power

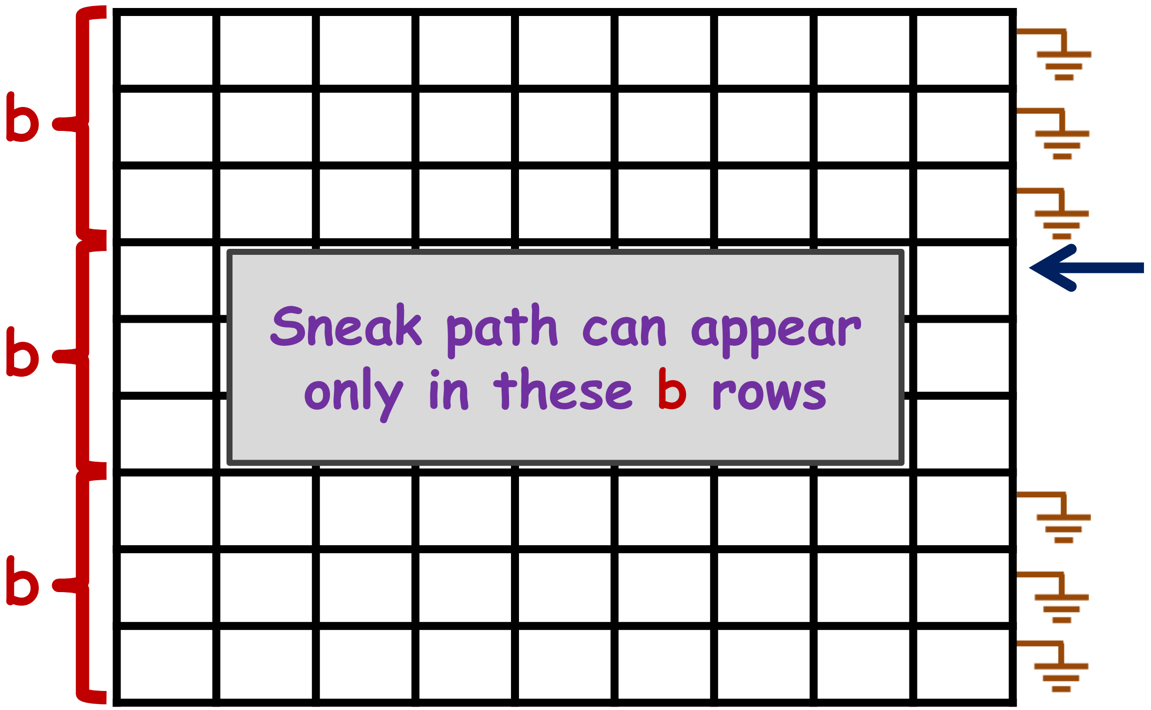
Poor capacity

A Mixed Solution



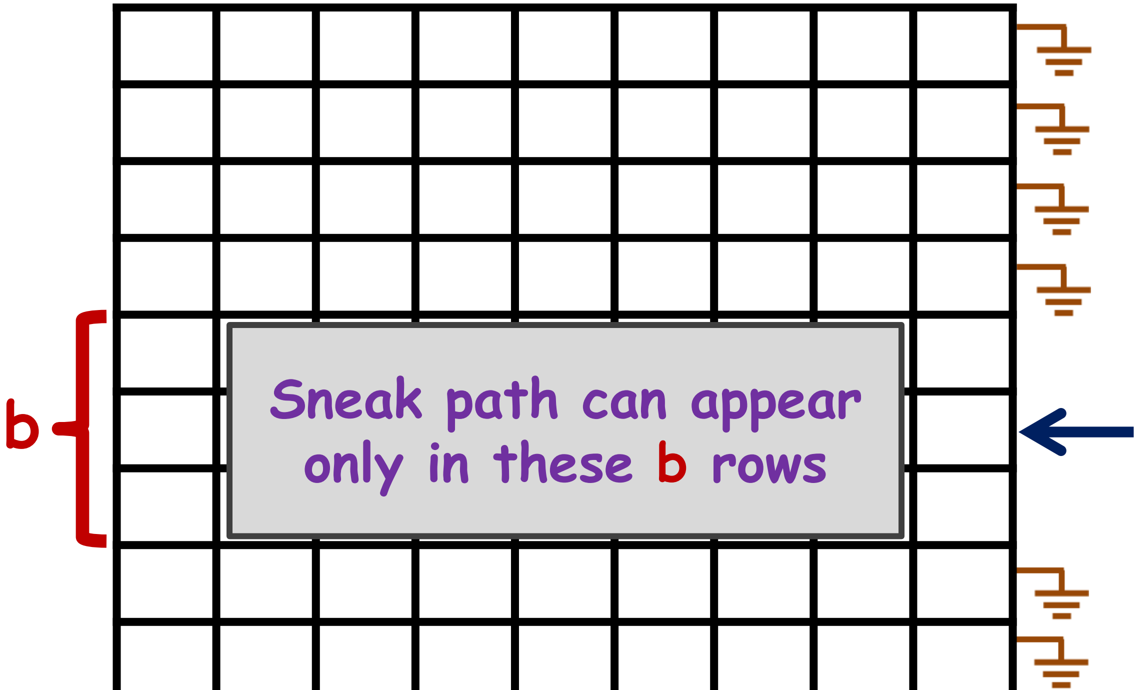
Grounding

- **Solution**: ground a smaller number of rows and combine with the coding solutions of the sneak path
- Two approaches to choose the grounded set
  1. Grounding **fixed subsets**



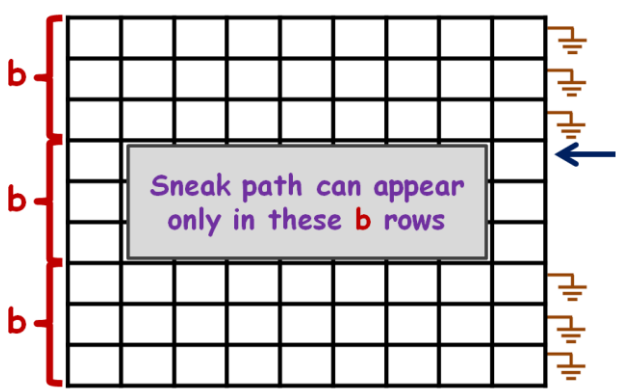
Grounding

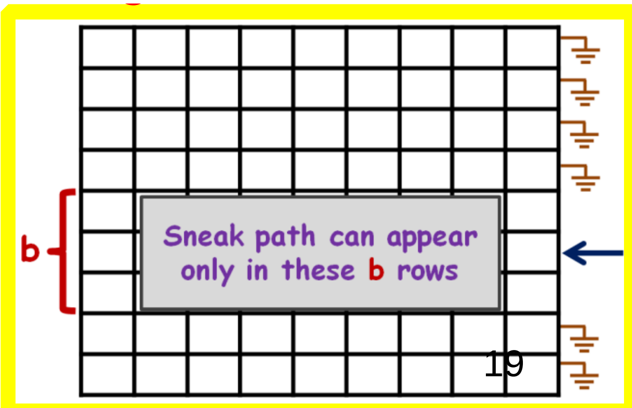
- **Solution**: ground a smaller number of rows and combine with the coding solutions of the sneak path
- Two approaches to choose the grounded set
  1. Grounding **fixed subsets**
  2. Grounding **around** the read row



Comparison

b	$\mathbb{C}_1(b) = \frac{\log(b+1)}{b}$	$\mathbb{C}_2(b) = \mathbb{C}_{\frac{b-1}{2},\infty}$
2	0.792	-
3	0.667	0.694
4	0.580	-
5	0.517	0.551
6	0.468	-
7	0.423	0.465

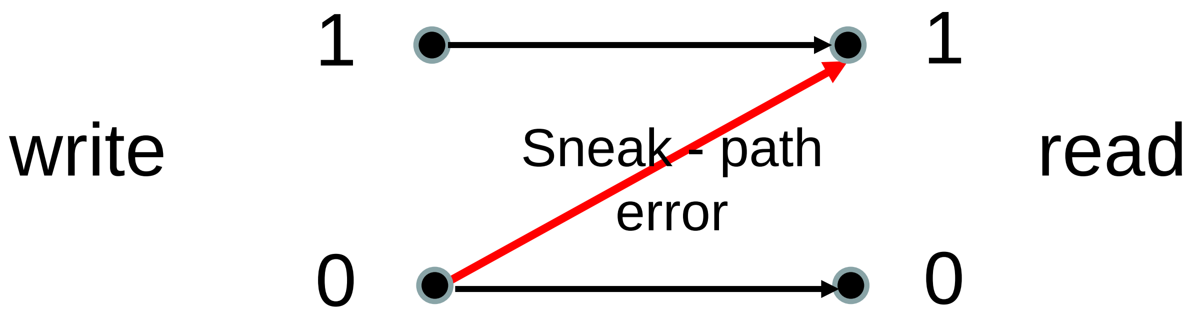




Part 2

Sneak paths as a **random error source**

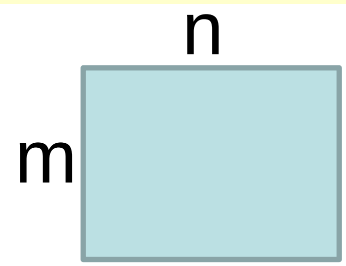
The Sneak-Path as Z Channel



Errors are deterministic given array values

0	1	0	0	0
0	0	0	1	0
1	0	0	1	1
0	1	0	0	0
1	0	1	1	0

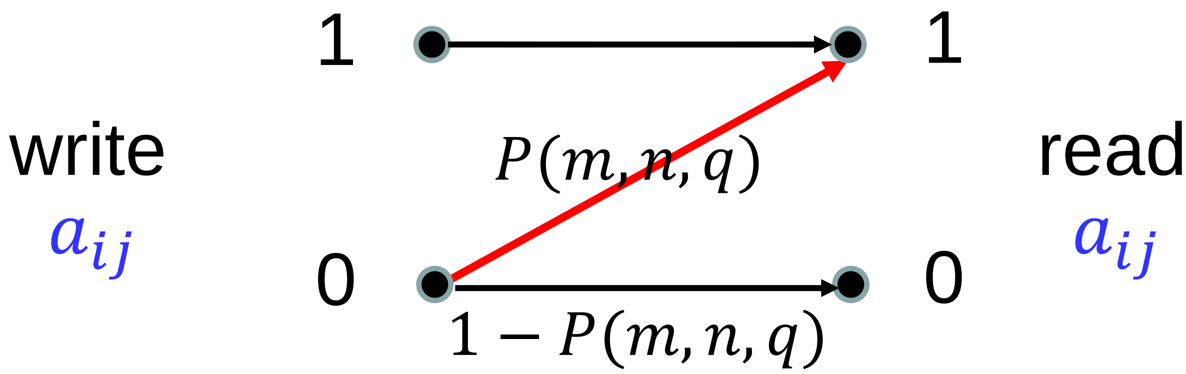
Sneak-Path Severity Factors



1. Array dimensions m,n
   – Large array → high vulnerability (more paths)

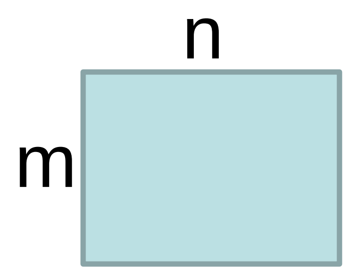
2. 0-1 bias q
   – More “1”s → more sneak paths
   – (All “1”s → no sneak paths)

The Sneak-Path Z Channel



Errors are probabilistic given array parameters:

1. Array dimensions m,n



2. 0-1 bias q

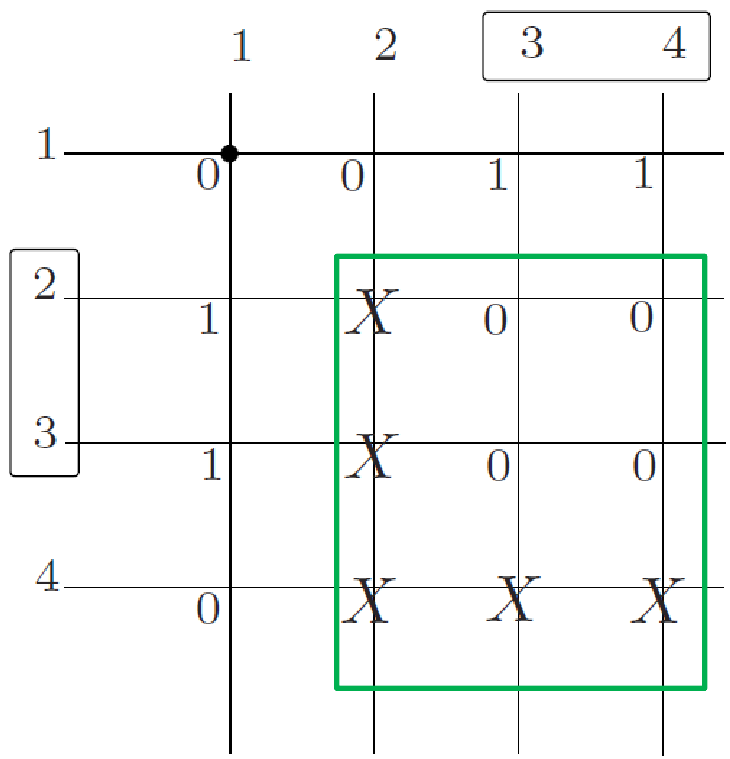
$$\Pr(a_{ij} = 1) = q \qquad \Pr(a_{ij} = 0) = 1 - q$$

The Transition Probability

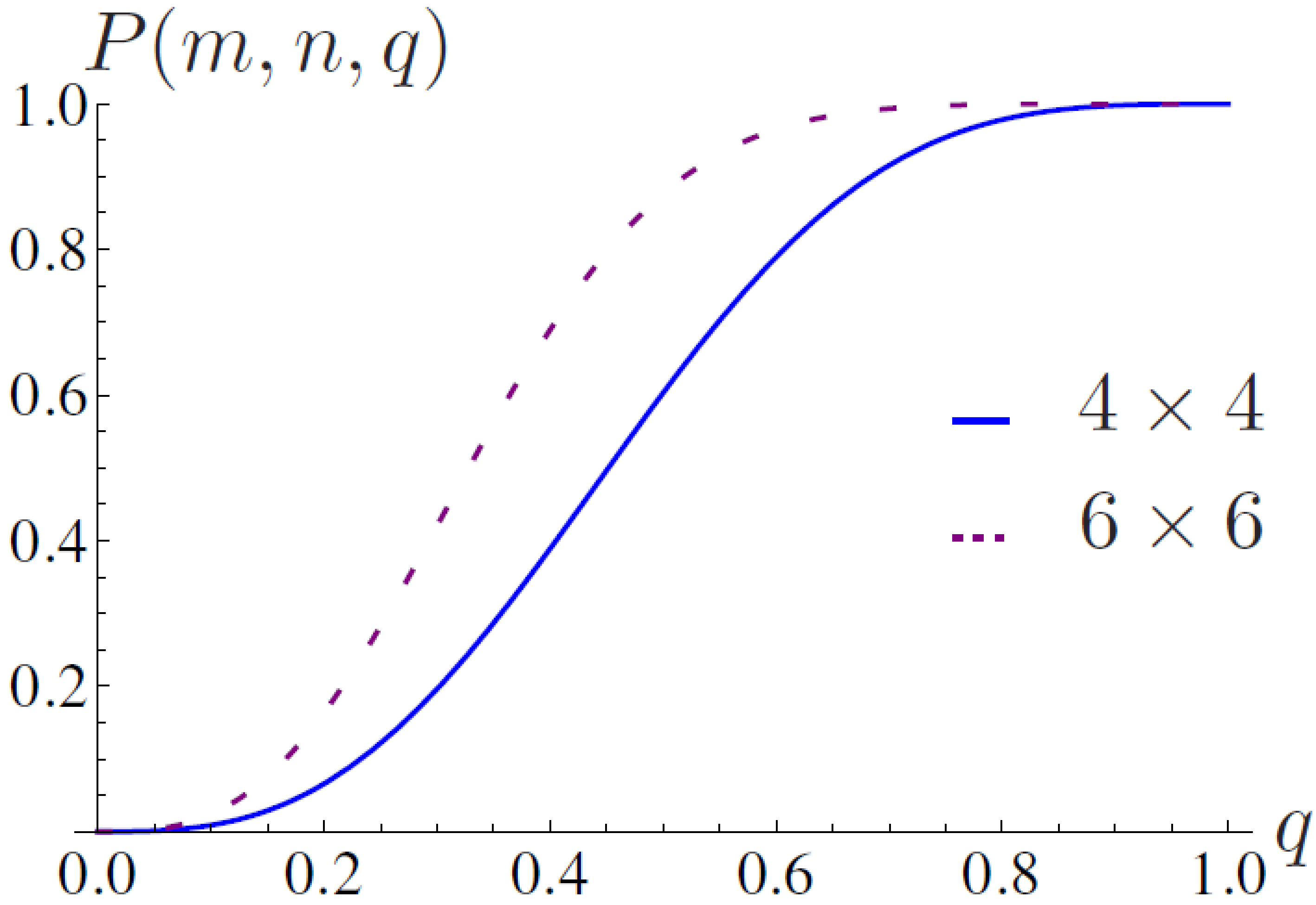
- Theorem:

$$P(m,n,q) = 1 - \sum_{u=0}^{m-1}\sum_{v=0}^{n-1}\binom{m-1}{u}\binom{n-1}{v} q^{u+v}(1-q)^{m-1-u+n-1-v+uv}$$

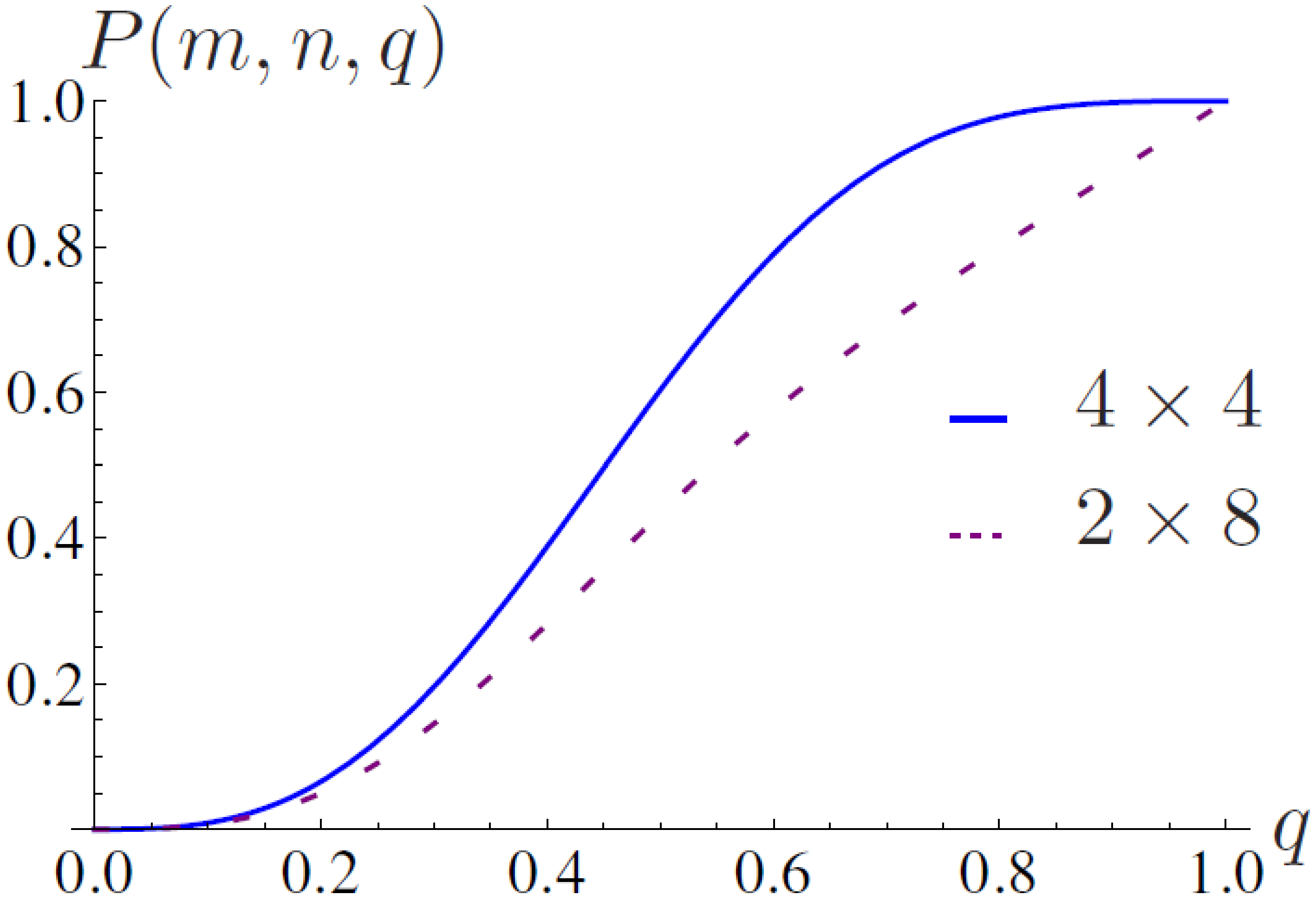
- Idea:



Transition Probability – Array Size



Transition Probability – Aspect Ratio



Joint-Probability Analysis

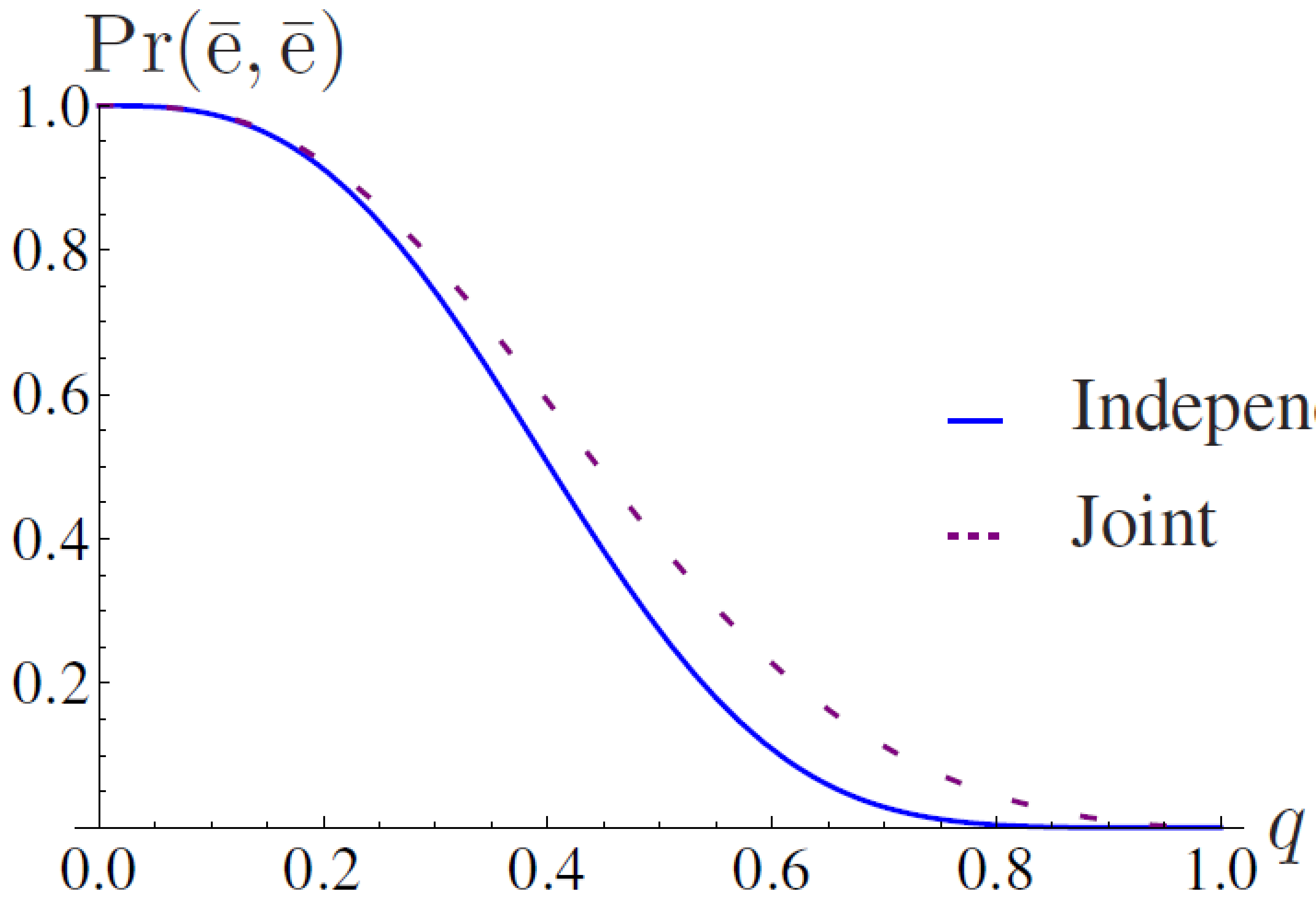
- Fact: sneak-path errors not independent within array
- Intuition: cells in same row/column share "1"s

- Theorem:

$$\Pr(\bar{e}_{i,j}, \bar{e}_{i',j}) =$$

$$\sum_{u=0}^{m-2}\sum_{v=0}^{n-1}\sum_{v'=0}^{n-1}\sum_{\sigma=0}^{n-1}\binom{m-2}{u}\binom{n-1}{v}\binom{v}{\sigma}\binom{n-1-v}{v'-\sigma}q^{u+v+v'}(1-q)^{m-2-u+n-1-v+n-1-v'+u(v+v'-\sigma)}$$

Joint-Probability: No Error



Joint-Probability: Error