Adaptive Threshold Read Algorithms in Multi-level Non-Volatile Memories

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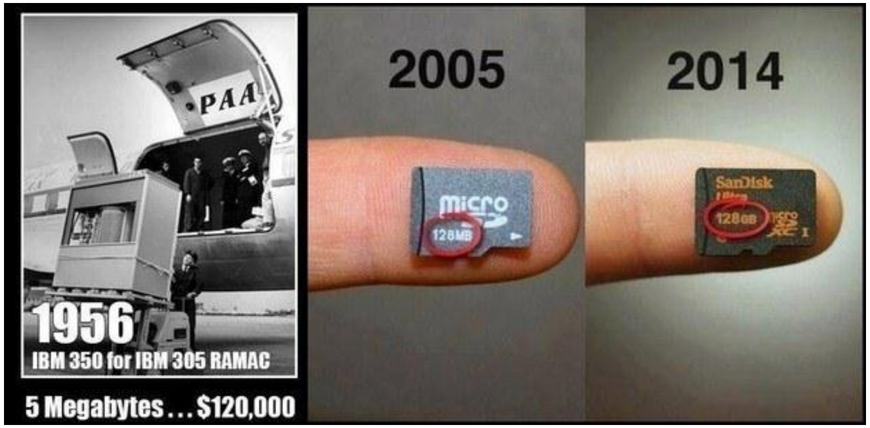
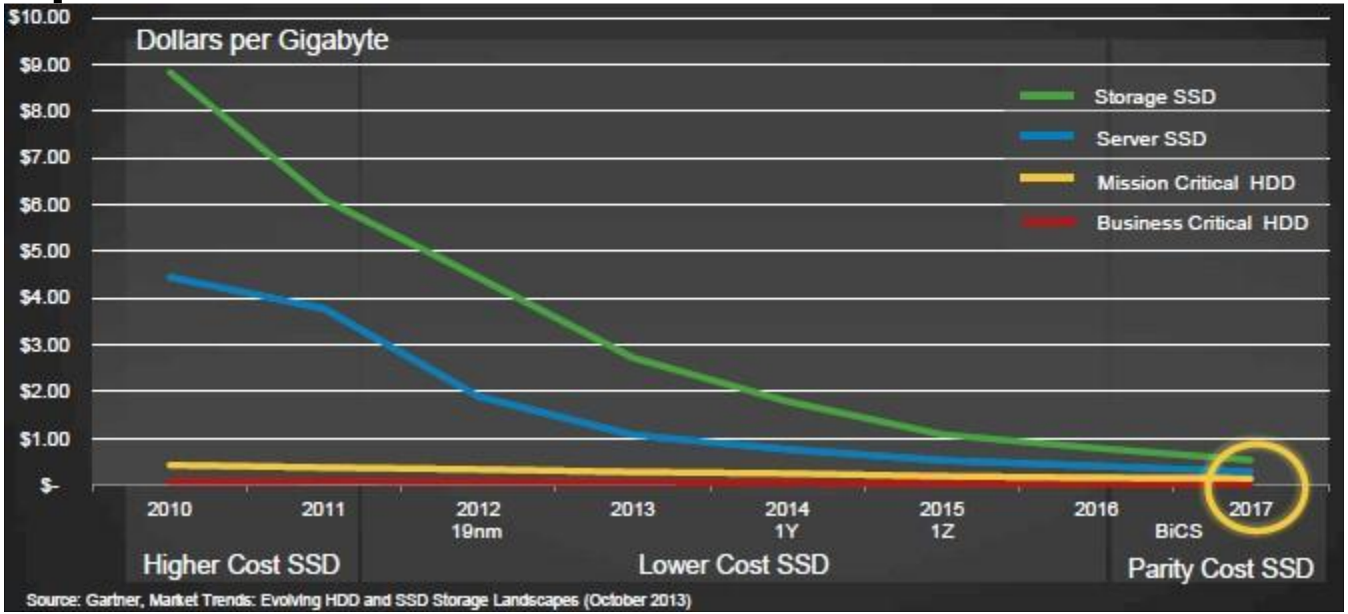
Introduction

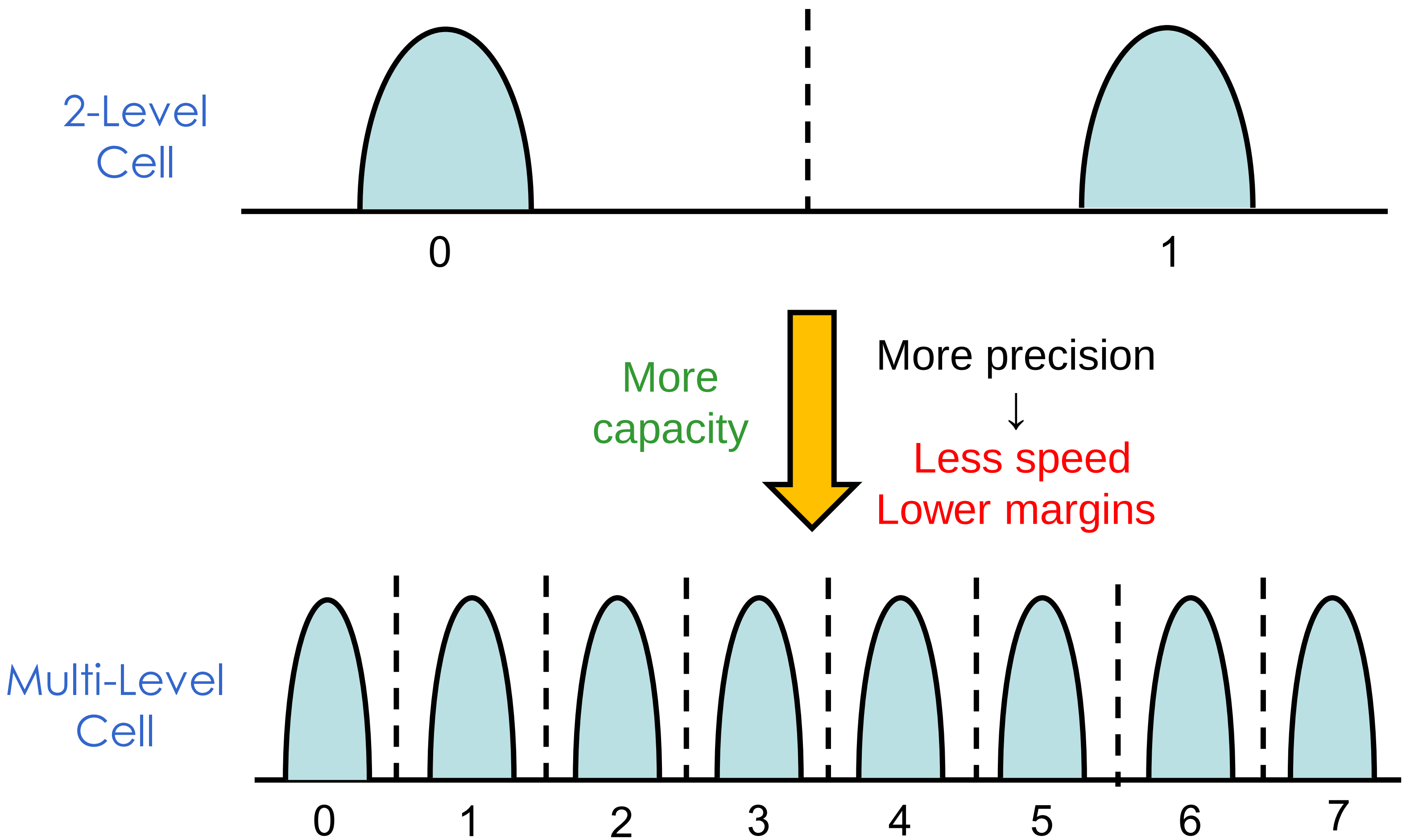
- In recent years



- Cost per GB reduced



Multi Level NVM

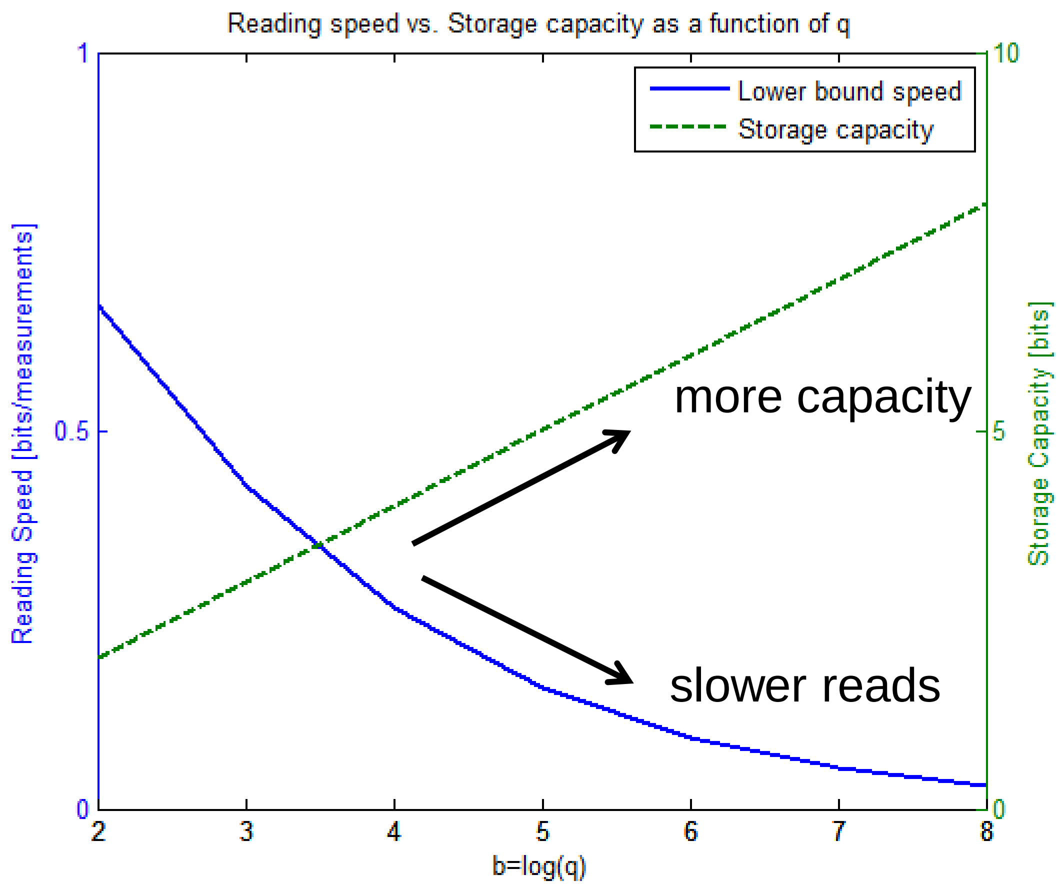


Scaling effects

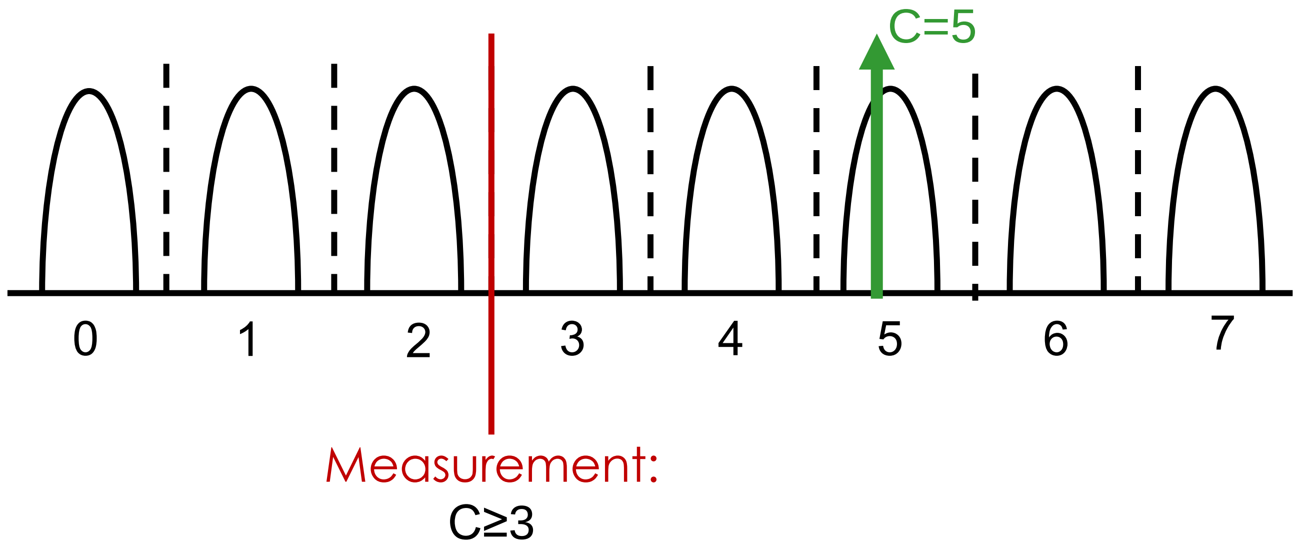
	SLC	MLC	TLC
Bits per cell	1	2	3
P/E Cycles	100,000	3,000	1,000
Read Time	25 µs	50 µs	~75 µs
Program Time	200-300 µs	600-900 µs	~900-1350 µs
Erase Time	1.5-2 ms	3 ms	4.5 ms

- **Increasing the number of memory levels:**
  - Increases density 
  - Decreases cost 
  - Increases read/write time 
  - Decreases lifetime 

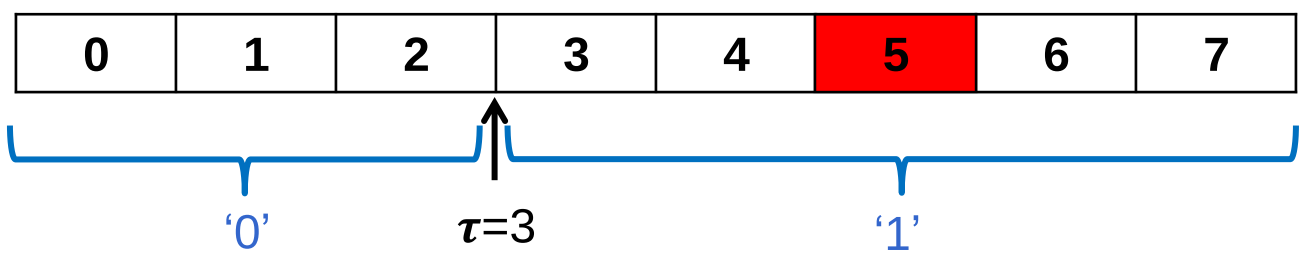
Reading speed vs. Storage capacity



Threshold Read



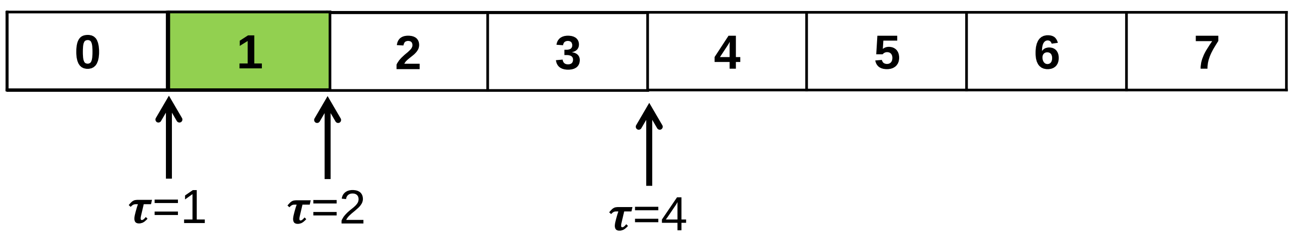
$$M_{\tau}(C) = \begin{cases} '0' & C < \tau \\ '1' & C \geq \tau \end{cases}$$



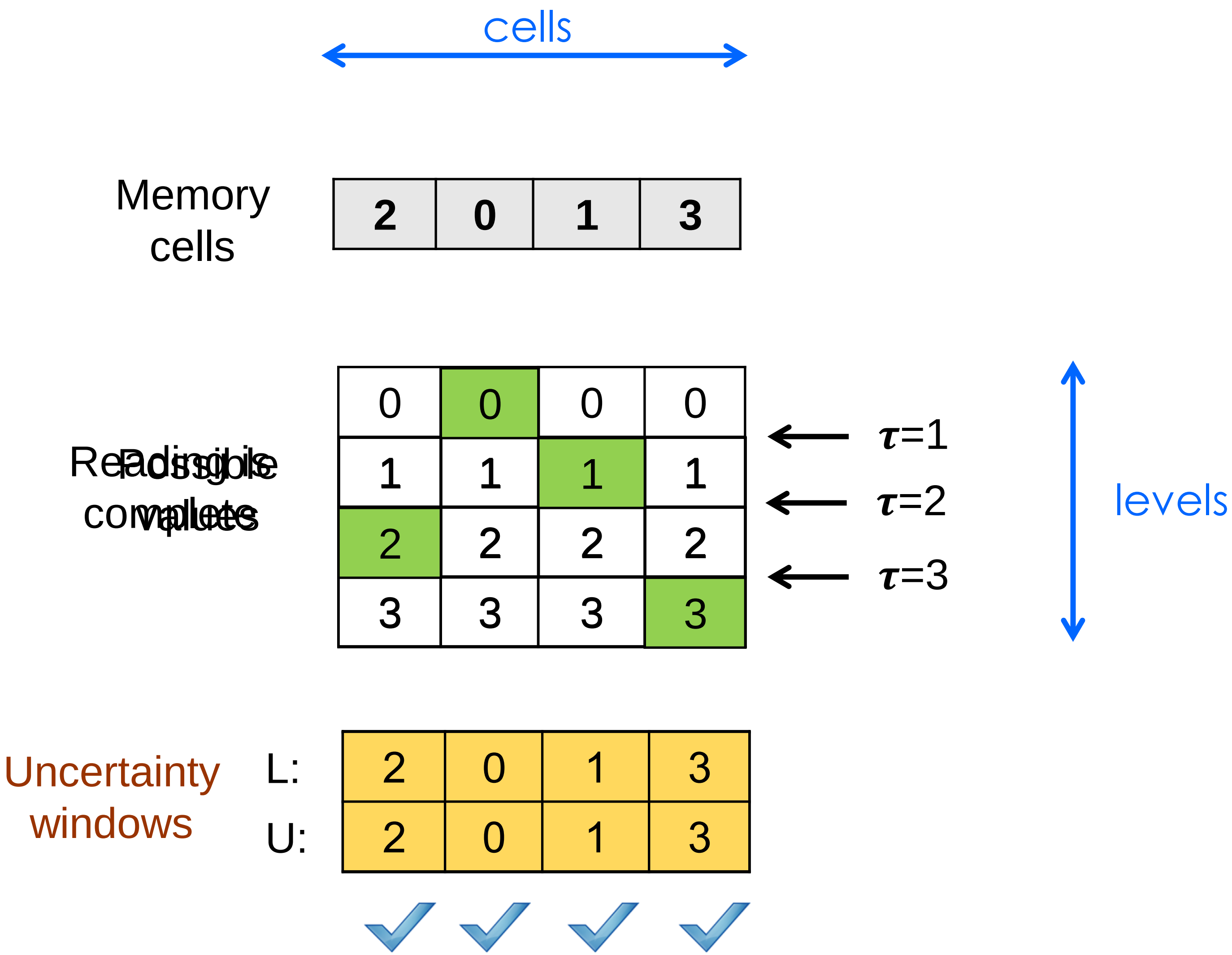
Threshold-Read Sequence

Possible values

True value



Parallel Threshold Read



Threshold Read Algorithms

Research question:

Given *n* cells with *q* levels, how many measurements are required to read all the cells completely?

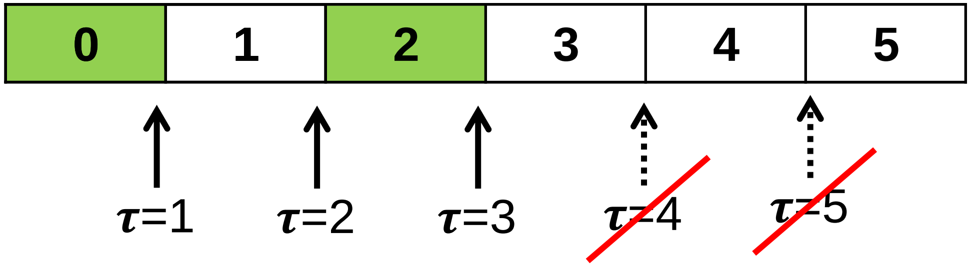
- **The reading is complete when L=U for all memory cells, e.g.**

L:	2	0	1	3
U:	2	0	1	3

- **Adaptive algorithms**
  - Next measurement depends on outcomes of previous ones
- **Average number of reads**
  - Assuming uniform level distribution, or other natural dist.
- **1< *n* << ∞**

1) Adaptive Sequential Scan

- **Measure from τ=1 to τ=q-1**
- **Stop when all *n* cell levels are determined**



Adaptive Sequential Scan - Analysis

- **The average number of measurements for adaptive sequential scan is given by:**

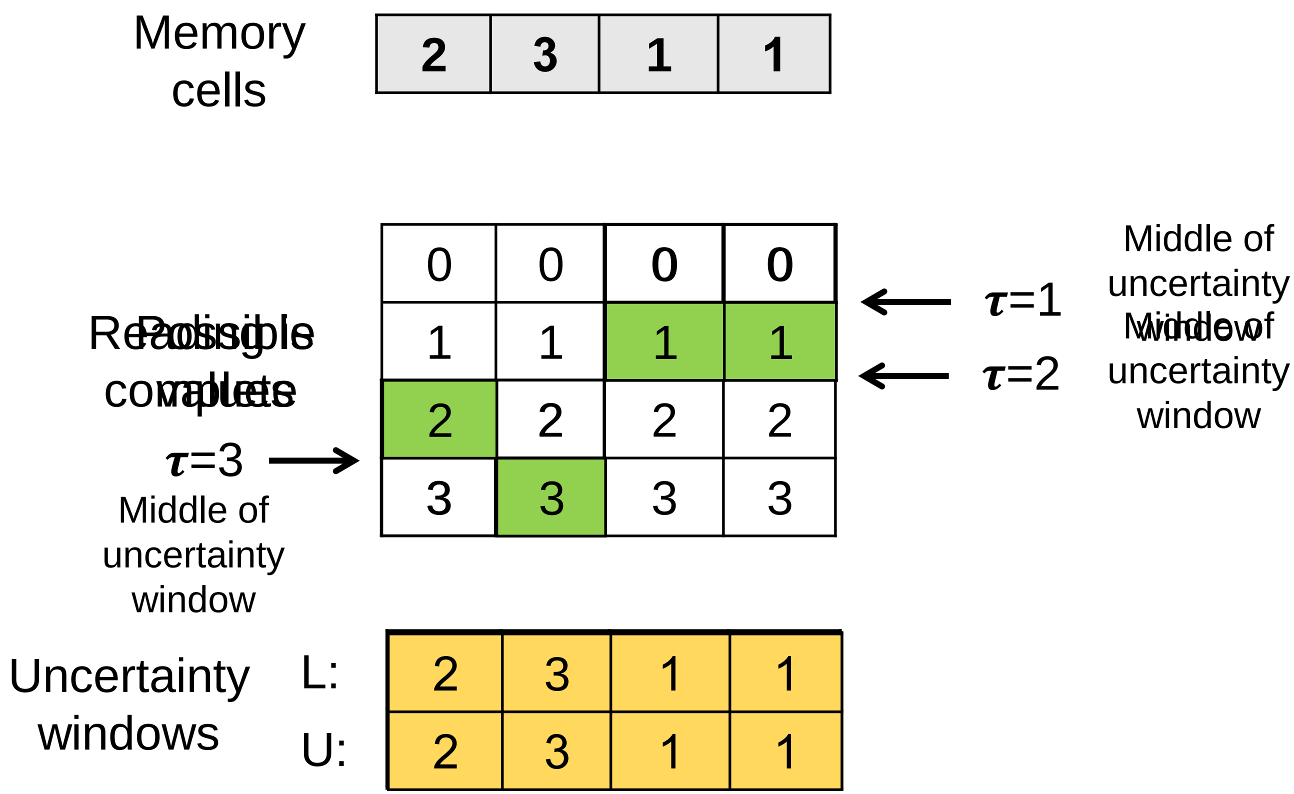
$$E[q-1-\#meas.] = \sum_{k=1}^{q-2}(q-1-k)\left[\left(\frac{k}{q}\right)^n - \left(\frac{k-1}{q}\right)^n\right]$$

- **After some algebra:**

$$T(n,q) = \underbrace{(q-1)}_{\text{non-adaptive seq. scan}} - \sum_{k=1}^{q-2}\left(\frac{k}{q}\right)^n$$

non-adaptive seq. scan

2) n-cell Binary Search



n-cell Binary Search Algorithm

1. Choose an uncertainty window [L,U] of a cell; measure $\tau = \frac{U+L+1}{2}$
2. For all cells $< \tau$: reduce uncertainty window to $[L, \tau\text{-}1]$
3. For all cells $\geq \tau$: reduce uncertainty window to $[\tau, \mathrm{U}]$
4. Return to 1 until L=U for all cells

Binary search – performance

- **The average number of measurements needed for binary-search read is given by the recursive formula:**

$$l = log_2 q$$

$$F(n,l) = \sum_{i=0}^{n} \frac{\binom{n}{i}}{2^n} \left(1 + F(i, l-1) + F(n-i, l-1)\right)$$

- **Where F(n,l)=0 if either n=0 or l=0**

- **An explicit analytic expression for F (n,l) is given by:**

$$F(n,l) = \sum_{k=0}^{l-1} 2^k \left[1 - \left(1 - \frac{1}{2^k}\right)^n\right] \xrightarrow[n \to \infty]{} q-1$$

3) Lower Bound

- **<u>Theorem:</u>**

Any read algorithm requires on average <u>at least</u> $LB(n,q)$ measurements given by

$$LB(n,q) = \frac{1}{q^n}\sum_{k=1}^{n} k! \cdot S(n,k) \cdot \sum_{L=1}^{k}\sum_{j=0}^{2} D_j(q,k,L)\cdot(k+L-j)$$

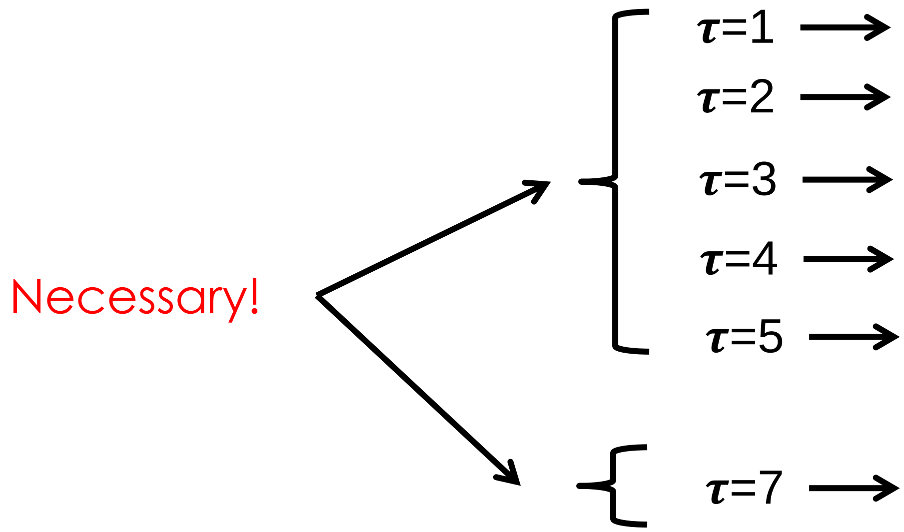
Explicit, known functions

Lower Bound – idea

- **Every level i used within the n cells requires measurements $\tau = i$ <u>and</u> $\tau = i + 1$.**

Memory cells

1	**3**	**4**	**7**

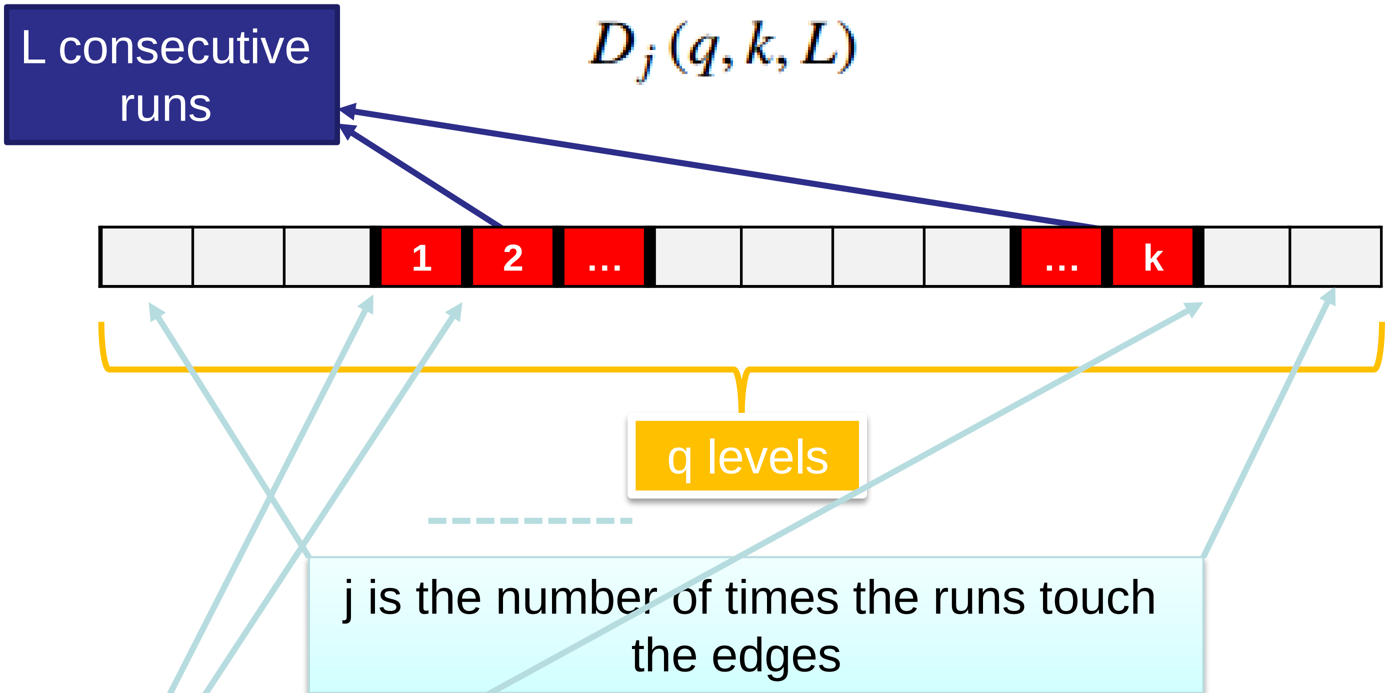


0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7

Incidence Sets

- **Definition:** given a vector of cell levels $\boldsymbol{c} = (\boldsymbol{c_1} \dots \boldsymbol{c_n})$ **with** $\boldsymbol{c_i} \in \{\boldsymbol{0} \dots \boldsymbol{q} - \boldsymbol{1}\}$ **we define:**
  - **Incidence set** as the set $I(c) = \{s \in \{1 \dots q-1\} | \exists i, c_i = s\}$
  - The **shifted incidence set** is defined as $I^*(c) = \{s \in \{1 \dots q-1\} | \exists i, c_i + 1 = s\}$
- **For a cell vector c, the number of measurements is at least** $|\boldsymbol{I}(\boldsymbol{c}) \cup \boldsymbol{I}^*(\boldsymbol{c})|$

The D_j functions



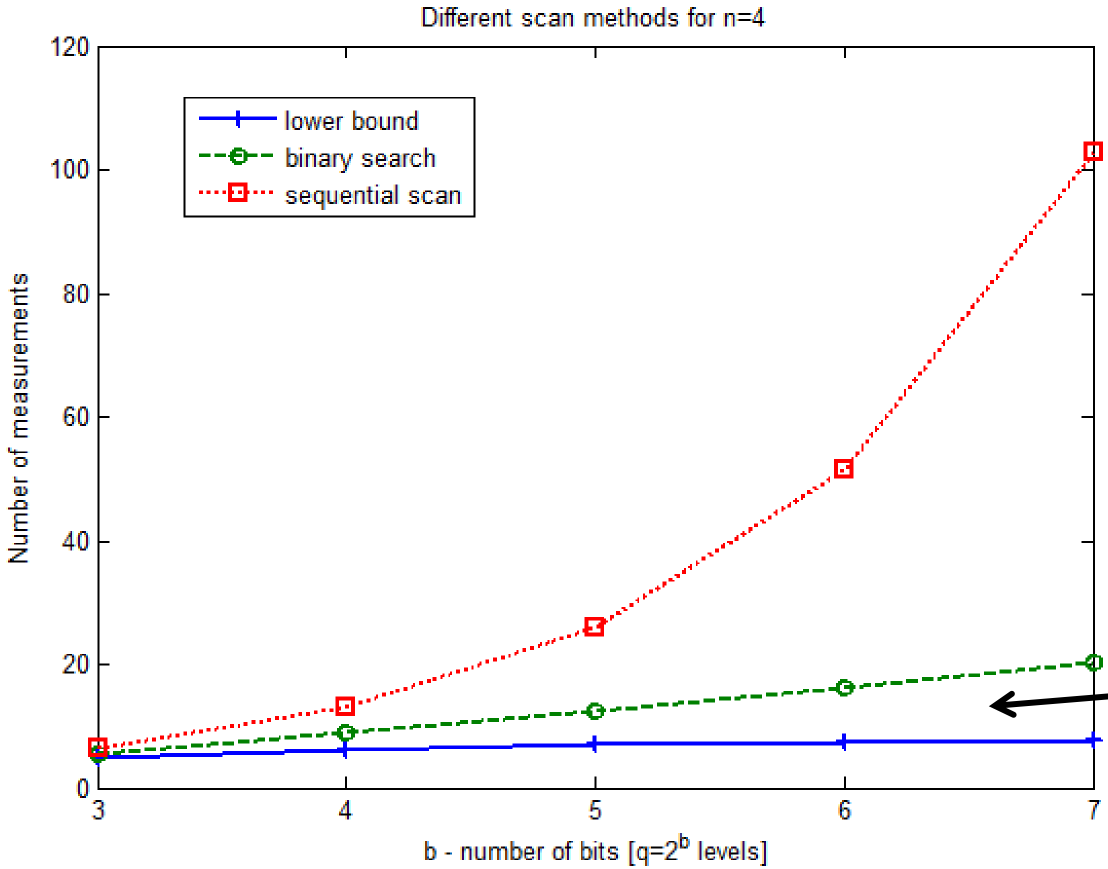
- **Each combination counted by $D_j(q,k,L)$ requires at least k+L-j threshold measurements**

Lower Bound – proof sketch

- Count incidence sets of each possible size, given k used levels
- Count level combinations with k used levels
- Average over k

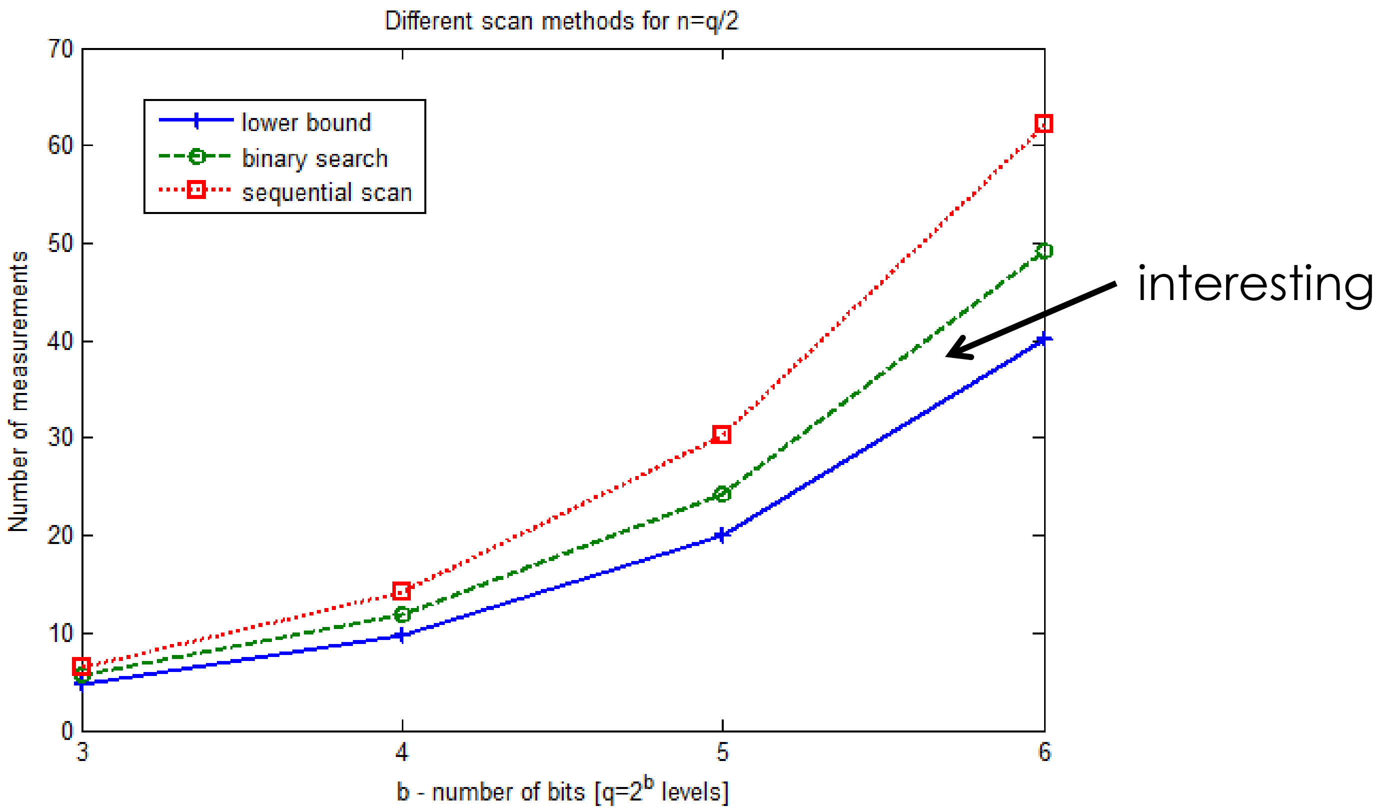
$$LB(n,q)=\frac{1}{q^n}\sum_{k=1}^{n}\boxed{k!\cdot S(n,k)}\cdot\boxed{\sum_{L=1}^{k}\sum_{j=0}^{2}D_j(q,k,L)}\cdot(k+L-j)$$

Analytic Results, n=4



interesting

Analytic Results, n=q/2



2D Read Algorithms

- **Cells in an $m \times n$ array**
- **2D Read algorithm:**
  1. Choose n cells to measure
  2. Choose level τ

2D Algorithms - motivation

- Suppose we read the following 2x2, q=8 array row-by-row (1D):

1	2
0	3

- Top row: at least 3 meas. {1,2,3}
- Bottom row at least 3 additional meas. {1,3,4}
- Total of 6 measurements for the array

2D Algorithms – motivation

- **Alternatively, <u>if we can choose</u> whether to measure a row <u>or</u> a column**

Original array

1	2
0	3

- Top row with $\tau = 2$

L:

0	2
0	0

U:

1	7
7	7

- Left column with $\tau = 1$

L:

1	2
0	0

U:

1	7
0	7

- Right column with $\tau = 3$ and $\tau = 4$ will reveal the entire array

- **2D reading reduced the number of measurements from 6 to 4**

Greedy 2D Algorithms

- **Select "best" n cells and threshold τ**
- **The criterion: minimize the sum of expected uncertainties after measurement.**

$$\Omega = \log(U - L + 1)$$

Expected uncertainty, cell i:

$$H_i(\tau) = \Pr(c_i < \tau)\log(\tau - L_i) + \Pr(c_i \geqslant \tau)\log(U_i - \tau + 1) =$$

$$\boxed{\frac{\tau - L_i}{U_i - L_i + 1}\log(\tau - L_i) + \frac{U_i - \tau + 1}{U_i - L_i + 1}\log(U_i - \tau + 1)}$$

The CRDF algorithm

- **The** *Column Row Degree of Freedom* **(CRDF) algorithm has the ability to choose in each scan whether to measure a row or column based entropy criterion**
- **The criterion is calculated for all rows and columns and for all possible thresholds**
- **The measurement is executed for the best threshold and row\column**

The ANDF algorithm

- **The** *Any N degree of Freedom* ***(ANDF) is very similar to the CRDF algorithm, however has the ability to choose any ‘n’ cells out of the 2D nxn array***
- **The ANDF is more flexible algorithm but it also very greedy and difficult to implement on hardware**
- **The ANDF serves as a practical lower bound**
- **The performance of both CRDF and ANDF was evaluated by simulations**

2D Lower Bound

- **<u>Theorem:</u>**

Given the uniform level distribution, a lower bound on the average number of *n*-cell measurements required to read an array of *N* cells is given by

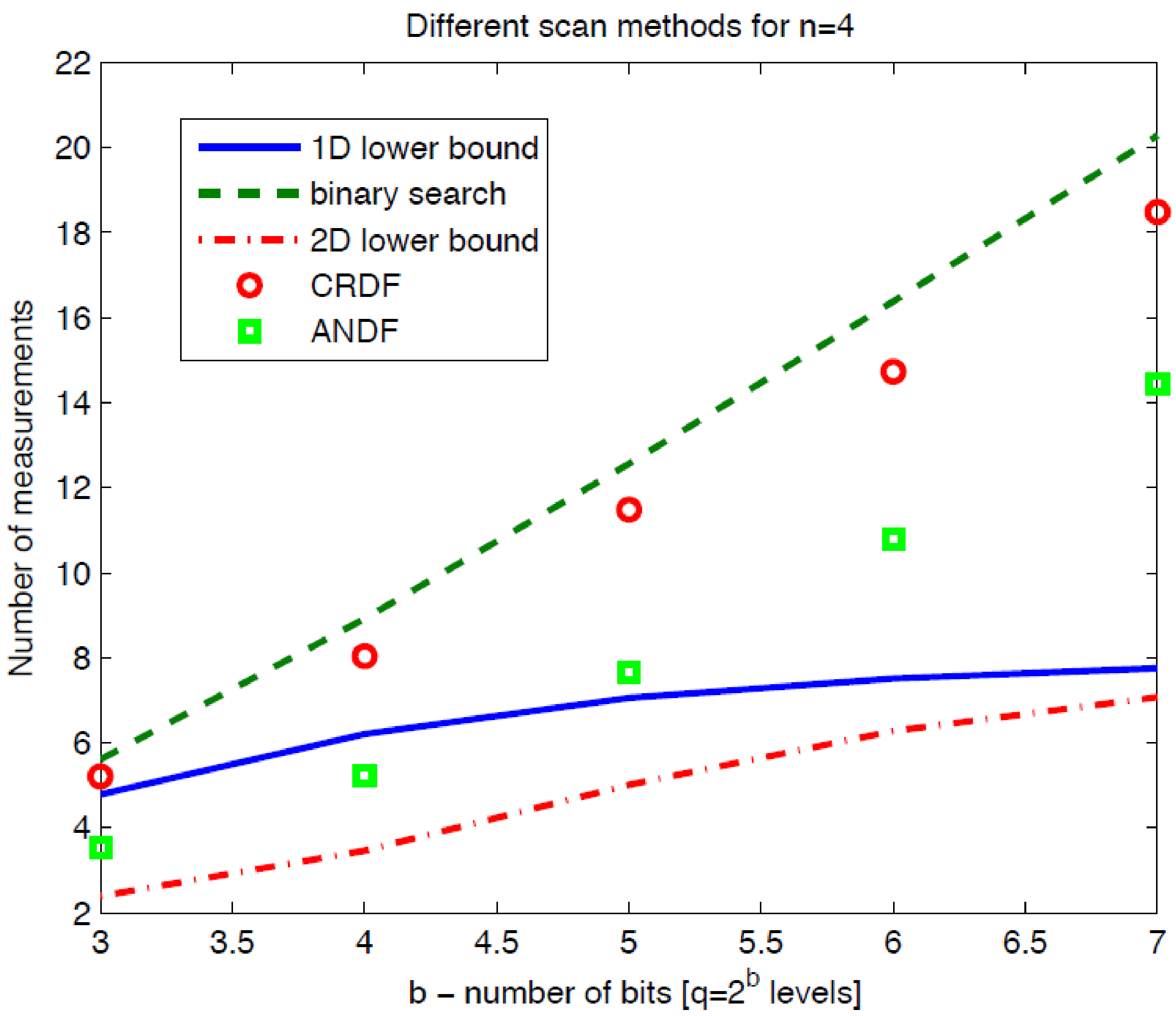
$$LB2D(N,q,n) = LB(N,q) + (q-1)\cdot\sum_{d=n+1}^{N}\left\lfloor\frac{d-1}{n}\right\rfloor\cdot\binom{N}{d}\cdot\left(\frac{2}{q}\right)^{d}\cdot\left(\frac{q-2}{q}\right)^{N-d}$$

The 1D lower bound

Selection Domains

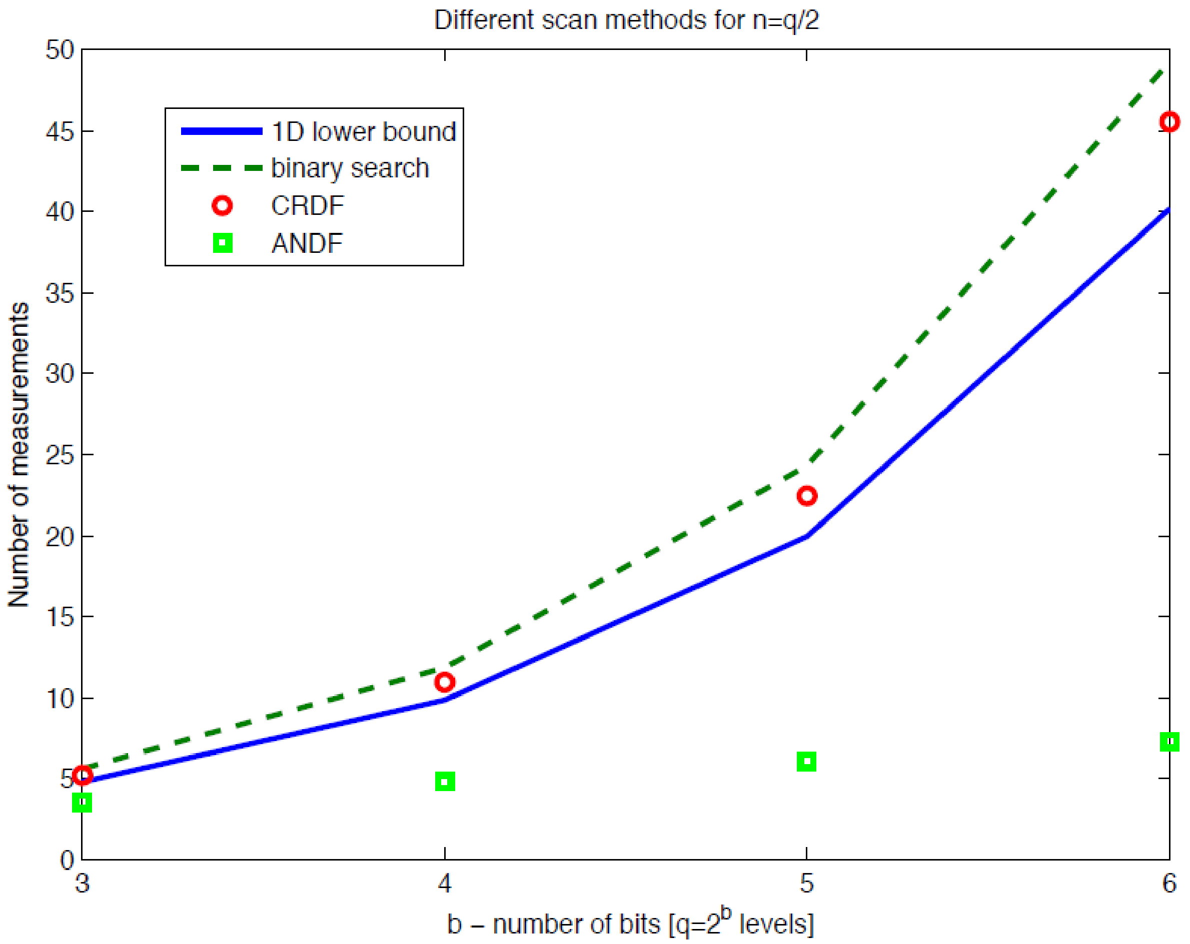
- **Only rows → binary search**
- **Rows or columns → CRDF algorithm**
  - column/row degrees of freedom
- **Any n → ANDF algorithm**
  - any n degrees of freedom
- **We also derived 1D & 2D lower bounds**

2D - Results



Different scan methods for n=4
Number of measurements
b – number of bits [q=2^b levels]
1D lower bound
binary search
2D lower bound
CRDF
ANDF
22
20
18
16
14
12
10
8
6
4
2
3
3.5
4
4.5
5
5.5
6
6.5
7

2D – Results cont.



Non-uniform Level Distributions

- **Prior bias toward level 0:**

$$\Pr(c=0)=\frac{m}{U+1}$$

  - For some real $m \geq 1$

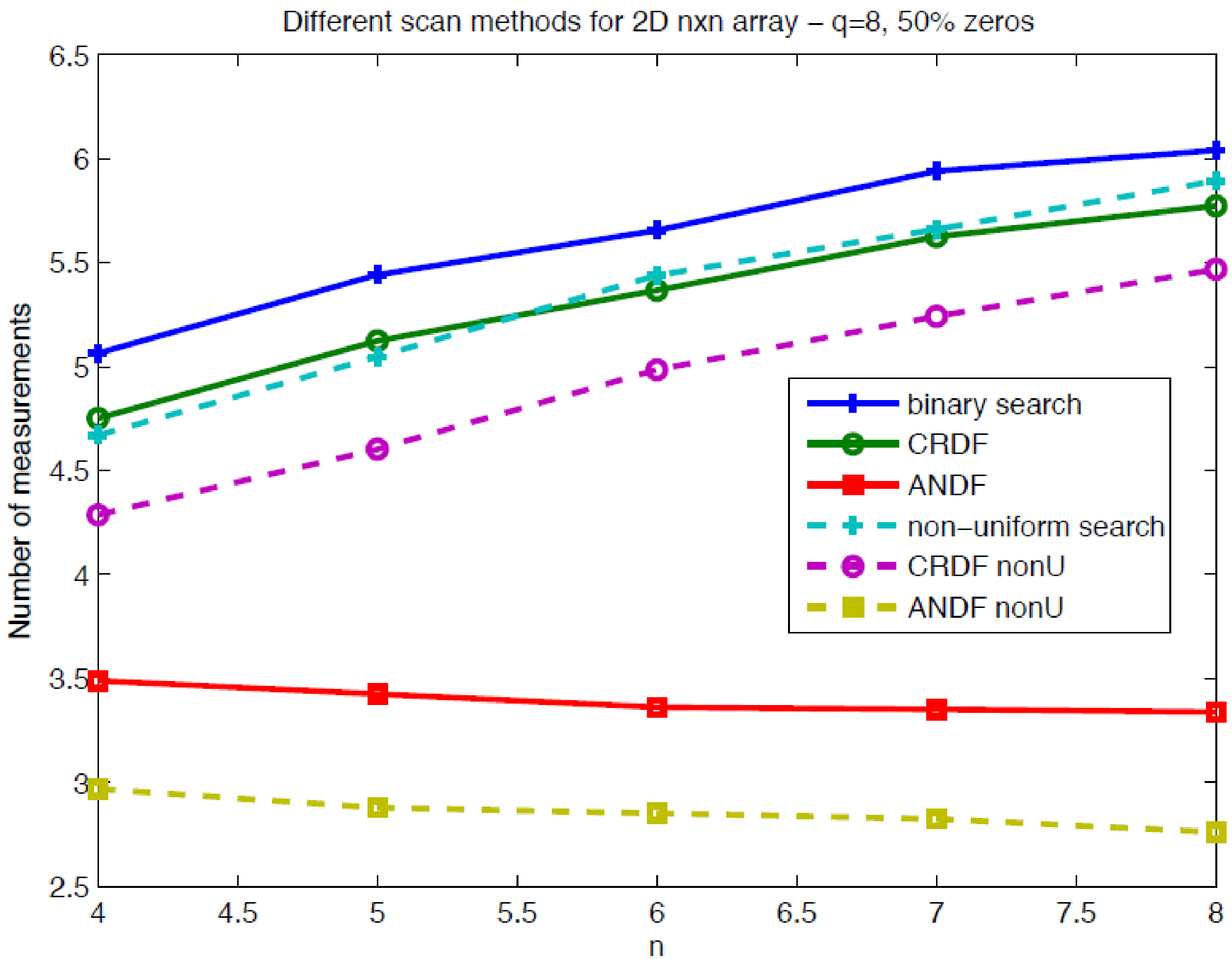
- **The remaining levels are distributed as:**

$$\Pr(c=k,k\neq 0)=\frac{U+1-m}{U\left(U+1\right)}$$

- **<u>Theorem</u>: the expected uncertainty after measurement of a cell with [0,U] uncertainty window is**

$$\begin{aligned} H(\tau) = {} & \frac{m}{U+1}\log\left(1+\frac{(U+1-m)(\tau-1)}{mU}\right) \\ & +(\tau-1)\frac{U+1-m}{U(U+1)}\log\left(\tau-1+\frac{mU}{U+1-m}\right) \\ & +\frac{(U+1-m)\left(U-\tau+1\right)}{U\left(U+1\right)}\log(U-\tau+1). \end{aligned}$$

Non-Uniform Distribution - Results



Conclusion

- Analysis of 1D read algorithms + derivation of a lower bound were presented
- We have also shown greedy 2D algorithms + a 2D lower bound

<u>Future work</u>:

- Combining coding in order to skip measurements